

# Hume's Central Principles

## *3. Hume's Logic: Relations, and Forms of Argument*



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# The Heart of Hume's Philosophy

- The core of Hume's "Chief Argument" is in *Treatise* Book 1 Part 3, the longest part of the entire *Treatise*.
- *Treatise* 1.3 is entitled "Of Knowledge and Probability", however:
  - Only *T* 1.3.1 deals with "Knowledge" (a word Hume uses in a strict sense, as meaning *deductive* knowledge).
  - Apart from the *title* of *T* 1.3.2, "probability" doesn't make an entrance until *T* 1.3.6.4.

# The Theme of Causation

- The real unifying theme of *Treatise* 1.3 is *causation* and causal reasoning. But Hume's route to his account is circuitous!
  - He starts (in *T* 1.3.1-2) by developing his taxonomy of relations into a theory of mental operations and of demonstrability;
  - He then focuses on the relation of causation, seeking the key impression;
  - On the way to this, he discusses the Causal Maxim, induction, probability ...

3(a)

Hume's  
Theory of  
Relations

A  
T R E A T I S E  
O F  
Human Nature :  
B E I N G  
An ATTEMPT to introduce the ex-  
perimental Method of Reasoning  
I N T O  
M O R A L S U B J E C T S.

*Rara temporum felicitas, ubi sentire, quæ velis ; & quæ  
sentias, dicere licet.*  
TACIT.

V O L. I.

O F T H E  
U N D E R S T A N D I N G.

L O N D O N :

Printed for JOHN NOON, at the *White-Hart*, near  
*Marcet's-Chapel*, in *Cheapside*.

M D C C X X X I X.

1739



# Introducing Relations

- Having explained the association of ideas, Hume calls it “a kind of **ATTRACTION**, which in the mental world” has remarkable effects like gravity in the physical world (T 1.1.4.6).
- One of these effects is to produce complex ideas by uniting simples together; these “may be divided into **RELATIONS**, **MODES**, and **SUBSTANCES**” (T 1.1.4.7).
- This provides a link into the main chapter on relations, T 1.1.5, though as we shall see, T 1.3.1 is also very important.

# Natural and Philosophical Relations

- *T* 1.1.5 starts with a distinction between two senses of the word “relation”. In one sense, we think of things as *related* when the idea of one *naturally* leads the thought to the other.
- So the “natural relations” are those that correspond to our associational tendencies – resemblance, contiguity, causation.
- But when philosophers talk about “relations”, they include *any kind of arbitrary* “subject of comparison”. Hume develops Locke’s taxonomy of such “philosophical relations”.

# Locke on the Types of Relation (1)

- Locke (II xxv-xxviii) emphasises:
  - “Cause and Effect” (II xxvi 1-2)
  - “Relations of Time” (II xxvi 3-4)
  - “Relations of Place and Extension” (II xxvi 5)
  - “Identity and Diversity” (II xxvii)
  - “Proportional Relations” (II xxviii 1)
- The last of these categories includes both what Hume calls “degrees in quality” and “proportions in quantity or number”.

# Locke on the Types of Relation (2)

- Locke then says there are “infinite others” of relations (II xxviii 1), notably:
  - “*Natural Relations*” such as “*Father and Son, Brothers ... Country-men*” (II xxviii 2)
  - “*Instituted, or Voluntary*” relations such as “*General ..., Citizen, ... Patron and Client, ... Constable, or Dictator*” (II xxviii 3)
  - Various moral relations (II xxviii 4-16)
- Note that Locke does not mean the same by “natural relation” as Hume.



# Locke to Hume on Relations (1)

- Locke's "diversity" apparently becomes Hume's "contrareity".
- Hume's "resemblance" – which he says enters into all relations – fulfils a similar role to Locke's 'agreement' (II xxviii 19).
- Locke doesn't treat "resemblance" as a single type, but recognises myriad forms of resemblance (e.g. "*Country-men, i.e. those who were born in the same Country*").



# Locke to Hume on Relations (2)

- Hume seems deliberately to subsume Locke's "natural", "instituted" and moral relations under cause and effect:

"... all the relations of blood depend upon cause and effect ..." (*T* 1.1.4.3)

"... the relation of cause and effect ... we may observe to be the source of all the relations of interest and duty, by which men influence each other in society, and are plac'd in the ties of government and subordination." (*T* 1.1.4.5)

# Locke and Hume on Relations

<i>[Locke doesn't speak of "agreement" as a relation]</i>	<i>Resemblance [a relation, but also involved in all relations]</i>
Cause and effect Natural, Instituted, Moral	Cause and effect
Relations of time Relations of place	Space and time
Identity	Identity
Diversity	Contrariety
Proportional relations	Proportions in quantity Degrees in quality

# Hume's Dichotomy

- Hume starts *T* 1.3.1 by dividing his seven types of relation into two groups (*T* 1.3.1.1):
  - The Four “Constant” Relations  
Those relations that “depend entirely on the ideas, which we compare together” (i.e. **resemblance, contrariety, degrees in quality, proportions in quantity or number**);
  - The Three “Inconstant” Relations  
Those relations that “may be chang’d without any change in the ideas” (i.e. **identity, relations of time and place, cause and effect**).

# A Taxonomy of Mental Operations

- Hume argues, rather simplistically, that his seven relations map neatly onto four different mental operations:
  - *resemblance, contrariety, and degrees in quality* are “discoverable at first sight” (*T* 1.3.1.2)
  - *proportions of quantity or number* are susceptible of demonstration (*T* 1.3.1.2-5)
  - *identity and relations of time and place* are matters of perception rather than reasoning (*T* 1.3.2.1)
  - *causation* is the only relation “that can be trac’d beyond our senses, [to] existences and objects, which we do not see or feel” (*T* 1.3.2.3)



	<i>Constant relations</i>	<i>Inconstant relations</i>
<b><i>Perception</i></b>	<u>Intuition</u> <ul style="list-style-type: none"> <li>■ <i>resemblance</i></li> <li>■ <i>contrariety</i></li> <li>■ <i>degrees in quality</i></li> </ul>	<u>Sensory Perception</u> <ul style="list-style-type: none"> <li>■ <i>identity</i></li> <li>■ <i>situations in time and place</i></li> </ul>
<b><i>Reasoning</i></b>	<u>Demonstration</u> <ul style="list-style-type: none"> <li>■ <i>proportions in quantity and number</i></li> </ul>	<u>Probability</u> <ul style="list-style-type: none"> <li>■ <i>causation</i></li> </ul>



# Hume's Dichotomy – the motive

- Hume gives his taxonomy of relations in order to facilitate his arguments:
  - That the Causal Maxim cannot be intuitively certain (*T* 1.3.3.2);
  - That relations of virtue and vice are not demonstrable (*T* 3.1.1.19).
- He seems to argue from the principle:
  - *Any proposition that is intuitively or demonstratively certain can contain only constant relations.*

# The Failure of the Dichotomy

- Sadly, this is nonsense. There are lots of “analytic” propositions involving identity, relations of time and place, or causation:
  - If  $A=B$  and  $B=C$ , then  $A=C$ .
  - Anything that lies inside a small building lies inside a building.
  - Every mother is a parent.
  - Anyone whose paternal grandparents have two sons, has an uncle.

# The Source of Hume's Mistake?

- I suggest that Hume confused, when considering propositions about objects:
    - Supervenience: *what is implied by the properties of the objects themselves (independently of their relative situation etc.)*
    - Analyticity: *what is implied by our ideas of the objects themselves (independently of ideas about their situation etc.)*
- (See Bennett 1971: 250-6 and 2001: 242-4 for the best published discussions of the issue)*

# Hume's Conceivability Principle

- Hume mostly relies not so much on his Dichotomy as on the *Conceivability Principle*:

“’Tis an establish’d maxim in metaphysics, *That whatever the mind clearly conceives includes the idea of possible existence*, or, in other words, *that nothing we imagine is absolutely impossible*.” (T 1.2.2.8)

“To form a clear idea of any thing, is an undeniable argument for its possibility, and is alone a refutation of any pretended demonstration against it.” (T 1.3.6.5)

“whatever we *conceive* is possible, at least in a metaphysical sense: but wherever a demonstration takes place, the contrary is impossible, and implies a contradiction.” (A 11, cf. E 12.28)



# Hume's Fork

- In the *Enquiry*, Hume replaces his Dichotomy with a distinction amongst propositions
  - Relations of Ideas can be known *a priori* – without any dependence on experience or real existence
    - by inspecting ideas; hence their falsehood is inconceivable and they are necessarily true.
  - e.g. Pythagoras' Theorem. (*E* 4.1)  
 $3 \times 5 = \frac{1}{2} \times 30$ . (*E* 4.1)  
All bachelors are unmarried.
  - The modern term is analytic (as understood e.g. by Ayer): “true in virtue of its meaning”.



# Matters of Fact

- Matters of Fact can't be known *a priori*, and their truth / falsity are equally conceivable:  
e.g.    The sun will rise tomorrow. (*E* 4.2)  
          The sun will not rise tomorrow. (*E* 4.2)  
          This pen will fall when released in air.
- Perhaps the closest modern term is synthetic: a proposition whose truth “is determined by the facts of experience” (Ayer, *LTL* 1971, p. 105).
- But Hume (like Ayer) presumes that the analytic/synthetic, a priori/a posteriori, and necessary/contingent distinctions all coincide.

# 3(b)

## Hume on Forms of Argument

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# The Four “Kinds of Evidence”

- Hume’s *Letter from a Gentleman* (1745) explains some background to his *Treatise*:
  - “It is common for Philosophers to distinguish the Kinds of Evidence into *intuitive*, *demonstrative*, *sensible*, and *moral*”
- “intuition” means *self-evidence*, “sensible” refers to *sensory* evidence.
- We have two forms of reasoning here, *demonstrative*, and “moral” or *probable reasoning*.

# Demonstrative and Probable

## ■ A Lockean distinction:

- In demonstrative reasoning, each link in the inferential chain is “intuitively” certain (hence = “deductive” in the modern non-formal sense).
- In probable reasoning, some links are merely probable (hence = “inductive” in a loose sense).

## ■ Hume takes over Locke’s distinction

- But in the *Enquiry* he also refers to *demonstration* as “reasoning concerning relations of ideas”,
- and to *probable reasoning* as “moral reasoning” or “reasoning concerning matter of fact”.



# Probable/Factual Inference

## ■ Consider:

Mars is red and round

*therefore*

Some round thing is coloured

- The premise and conclusion are matters of fact, so is this “reasoning concerning matter of fact”?
  - Is the inference merely “probable”? No!
  - Does it go beyond “relations of ideas”? No!
  - Does justifying the inference require any appeal to experience or to causal relations? No!
  - Hence Hume would have to count it as demonstrative.



# Demonstration = Deduction?

- So deductive arguments – even those with matter of fact premises and conclusions – must count as “demonstrative” for Hume.
- But this is controversial, if view of passages such as the following:
  - “no matter of fact is capable of being demonstrated” (*T* 3.1.1.18);
  - “It seems to me, that the only objects of the abstract sciences or of demonstration are quantity and number ...” (*E* 12.27, cf. *T* 1.3.1.5).

# 'No Matter of Fact is Demonstrable'

- Suppose I claim to *demonstrate* that all crows are black.
  - Ridiculous, you would say! How can I possibly demonstrate such a contingent claim?
  - “Well”, I reply, “here’s my demonstration”:
    1. All crows are birds.
    2. All birds are black. $\therefore$  All crows are black.
  - That’s a demonstrative argument, isn’t it?

# What is Demonstrated?

- The crows argument is indeed *demonstrative*, but that isn't enough to make it a *demonstration* of its conclusion.
- To demonstrate *Q from P* is not the same as demonstrating *Q tout court*. The latter requires that the argument's premises are known with certainty to be true.
- Hume denies that any matter of fact can be *demonstrated (tout court)*. He nowhere denies that one matter of fact can be demonstrated *from another*.

# Is Demonstrative Reasoning Limited to Mathematics?

“There remain, therefore, algebra and arithmetic as the only sciences, in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty.” (*T* 1.3.1.5)

“It seems to me, that the only objects of the abstract sciences or of demonstration are quantity and number ...” (*E* 12.27)

- But Hume’s account of this limit is in terms of the *relative clarity* of mathematical and moral ideas.
- So if we want to find *a posteriori* demonstrative arguments of any complexity, we have to look to applied mathematics ...



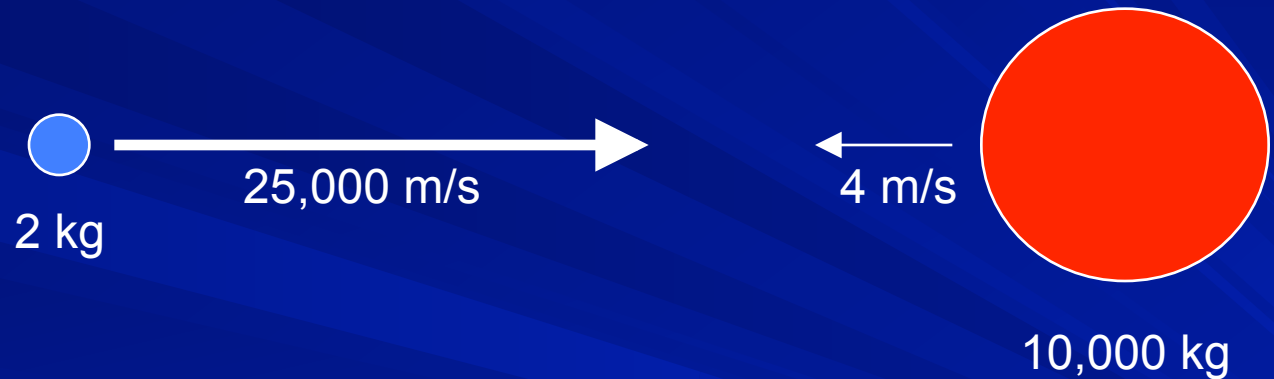
# Hume on Applied Mathematics

- Hume's most explicit discussion of "mixed mathematics" is in *Enquiry* Section IV:

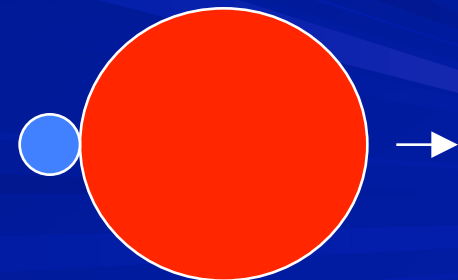
"it is a law of motion, discovered by experience, that the moment or force of any body in motion is in the compound ratio or proportion of its solid contents and its velocity; and consequently, that a small force may remove the greatest obstacle ... if, by any contrivance ... we can encrease the velocity of that force, so as to make it an overmatch for its antagonist." (E 4.13)

- The momentum of a body is equal to its mass multiplied by its velocity.
- In any collision the total momentum of the colliding bodies (in any given direction) is conserved.

*Before ...*



*After ...*



“Geometry assists us in the application of this law ... but still the discovery of the law itself is owing merely to experience, and all the abstract reasonings in the world could never lead us one step towards the knowledge of it.” (*E* 4.13)

“Mathematics, indeed, are useful in all mechanical operations ... But 'tis not of themselves they have any influence. ... Abstract or demonstrative reasoning ... never influences any of our actions, but only as it directs our judgment concerning causes and effects.” (*T* 2.3.3.2)

# Demonstration = Deduction

- Hume clearly does accept the possibility of demonstrative argument in applied mathematics; hence he cannot be restricting demonstration to the *a priori*.
- The natural interpretation of Humean “demonstration” – especially in the light of Hume’s Fork – is therefore “deduction” (in the informal sense: an argument whose premises guarantee its conclusion).



# Locke versus Hume on “Probable Reasoning”

- Although Hume follows Locke in taking for granted a general distinction between demonstrative [deductive] and probable [inductive] reasoning, the two differ profoundly regarding the nature of the latter.
- Locke sees the operation of reasoning – both demonstrative and probable – as involving the *perception* of evidential connexions.
- Hume denies any such perception in the case of probable [inductive] inference.

# Locke on Reason as *Perception* (1)

“we ... looke for noe greater certainty then what our eyes can afford us, the whole evidence of this assurance being noe more then what the word *Demonstration* doth naturally import; which is to shew any thing as it is & make it be perceived soe that in truth what we come to know this way is not by prooffe but intuition, all the prooffe that is used in this way of knowledg being noe thing else but shewing men how they shall see right ... without useing arguments to perswade them that they are soe”

(*Draft B* of Locke's *Essay*, 1671, p.153)

# Locke on Reason as *Perception* (2)

*“Inference ... consists in nothing but the Perception of the connexion there is between the Ideas, in each step of the deduction, whereby the Mind comes to see, either the certain Agreement or Disagreement of any two Ideas, as in Demonstration, in which it arrives at Knowledge; or their probable connexion, on which it gives or with-holds its Assent, as in Opinion. ... For as Reason perceives the necessary, and indubitable connexion of all the Ideas or Proofs one to another, in each step of any Demonstration that produces Knowledge; so it likewise perceives the probable connexion of all the Ideas or Proofs one to another, in every step of a Discourse, to which it will think Assent due. ...”* (Essay IV xvii 2).

# “Nothing But a Species of Sensation”

- Contrast Locke’s view of probable reasoning with what Hume at *T* 1.3.8.12:

“Thus all probable reasoning is nothing but a species of sensation. ’Tis not solely in poetry and music, we must follow our taste and sentiment, but likewise in philosophy. When I am convinc’d of any principle, ’tis only an idea, which strikes more strongly upon me. When I give the preference to one set of arguments above another, I do nothing but decide from my feeling concerning the superiority of their influence.”