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Contributor So shall we begin?

This is a very special subject right. Quantum mechanics is quite unique in your undergraduate training in the sense that it is the piece of physics which is – it is the great intellectual accomplishment of the last century.

It is the piece of physics which is least understood. Relativity and so on is perfectly clear and tied up. Of course there is a theory of everything or whatever on the frontier of the subject. But here we have a piece of undergraduate physics which is universally agreed is not properly understood in its deepest underpinnings.

It's still fundamentally mysterious and it's also quite extraordinarily specific to physics. And yet it completely underpins – it's absolutely fundamental. You cannot understand anything about your body, the table, the sun, anything. Everything is a manifestation of quantum mechanics in a very direct way.

So it's an extraordinarily exciting subject and I think it's – what's particularly exciting for you should be that there are pieces of this still to be put in place. There are mysteries here still to be resolved. And I think there's no reason why the person who does that, or people who contribute to doing that, should not be here in the audience today.

I don't think it's going to be done by people of my generation. It looks as if it's something that still needs to be done by people coming along with a fresh look at it.

So it's extraordinarily worth working out. Physics isn't easy. And quantum mechanics is one of the hard bits of physics. This course isn't easy. But it's extremely well worth working at and I would stress the importance of working on the problems.

There are masses of problems, right. Too many problems. There maybe, you will think when you look at the problem sets which are just some abstract of the problems, there are already too many problems.

But it is the way to learn and understand about physics is to work with the apparatus and

think about the meaning of the solutions that you get and so on and so forth.

And so I really would urge you to work as hard as you can at those problems and the reason for providing solutions to many of the problems is because you would – even if you can solve the problem yourself, you might find it interesting to see how I solved the problem. You know and develop your technique in that way.

Of course quantum mechanics has a very funny way of looking at the world. That's part of the problem. And it's by constant practice and experience that you'll deepen that understanding.

Okay, so.

Einstein, as everybody knows, didn't like quantum mechanics. But I think the reason why he didn't like quantum mechanics, which he expressed as "God doesn't play dice", was not a good reason. There may be – it may be that one shouldn't like quantum mechanics but that's not a good reason.

Let's just think about that for a moment. Quantum mechanics , sorry, physics is about predicting the future. It's about saying what's going to happen. If you lean a ladder up against a wall, you would like to know if you tread on the ladder whether it's going to slip and fall down. That kind of thing.

And if the data on which we work are always uncertain and the systems with which we work are never isolated and our theory crudely always applies to something which, for which we – physics is an apparatus where if you put in certain statements about what the system is – for example, with the ladder, what the roughness of the ground is, what the roughness of the wall is, what the weight of the ladder is, and so on and so forth, if you describe the system accurately, then you will get a precise prediction out.

But in the real world, not only can you not – there's always uncertainty in the data. You can't say exactly how rough the floor is because the roughness of the floor varies from place to place. You're not quite sure where you put down the ladder and so on. So the data that you're working with are uncertain.

So, what you should really do – what the best that you can actually do if you really want to push yourself to the most precise results is to write probability distributions. You can say that the probability of the ladder slipping from this position is such and such. The probability of a ladder slipping from that position is such is such.

In simple cases, you have a very sharp – you have a very narrow range of probabilities. The probability in certain positions is almost one that it won't slip. In other places it's almost one that it will slip. And so we can give a simple answer and we say, well, the critical angle for it slipping is 43 degrees and 34 minutes or whatever else. But if you really, really, really want to know something accurately, if you really want to push your predictions to the extreme or as hard as you can, you will have to calculate a probability distribution.

And calculating probability distribution is hard. In classical physics, it's hard. We will find that in quantum mechanics it's actually rather easier to calculate probability distributions with the quantum mechanical apparatus than it is with the classical physical apparatus.

Which is just as well because in quantum mechanics we're working on a theory. It arose

out of attempts to understand things that are so small that they are always seriously not isolated. So an electron carries a charge. Consequently, it is always in contact with – it's always interacting with the electromagnetic field.

But the electromagnetic field is, it turns out, always quivering. It's – so we never know what the electromagnetic field – even under the most precise control of electromagnetic field – you put your electron inside some resonant cavity, you cool the resonant cavity as close to absolute zero as you can and so on. No matter how hard you work, it turns out that electromagnetic field is in an unknown configuration.

Consequently, your electron is subject to uncertain disturbances. Consequently, what the electron is going to do, the best you can do is predict probabilistically in the same sense that when the horses are racing at Sandown Park or whatever, the results are going to be probabilistic. You know don't what a particular horse is going to do on a particular day because of all the – it's not an isolated system. It may have eaten something it didn't approve of at breakfast that morning, etc.

So, so it is natural that we should be working with probabilities. It is natural that the calculation – so whereas in classical physics, you, when you're talking about a cricket ball or a shell shot out of a Howitzer, you operate under the fiction – which is a very good fiction – that at every point in the trajectory, at every time, every precisely measured time, the shell – the centre of mass of the shell – has a very precise co-ordinates. And these co-ordinates progress in a very accurately calculable way.

You have only one number to calculate – well three numbers, I suppose: the x, y and z co-ordinates of the shell. To calculate it each time, you don't have to calculate a probability distribution. Well, in a simple case, you don't.

If you're considering what will happen when an electron leaves an electron gun, because of the quivering electromagnetic field, whatever uncertainty there was in the configuration of the electron before it shot out of the gun, and so on, it's inevitable that you're calculating a probability distribution for where the electron is going to go.

And calculating a probability distribution for every possible value of x is clearly going to be a hell of a lot more work than calculating one particular value of x. Right. So that's why it's going to be mathematically complex. And why it's going to involve probabilities and let's just remind ourselves of some basic facts about probabilities which I think, is this correct that in Professor Blundell's course, he's already talked about the laws of probability? Yes, good.

So the things we need to just remind ourselves of is that if we've got two independent events, the probability that we get the event A and the event B, is going to be the product of PA and PB. So we multiply the probabilities of independent events such as that, if you throw two dice, the probability that one die comes up with number one and the other comes up with number 6, so PA might be the probability that the first die, the red one, comes up with number one and PB might be the probability that the black die comes up with a number six.

Then this is the probability that the red one comes up with one – what did I say, six? Whatever. This is the probability of that particular configuration. And you get a product.

And the other rule that's important for us is that the probability of A or B is equal to PA + PB, if they're exclusive events. So that's the probability of if I throw a single die, that I get either a one or a six. Because I can't get both a 1 and a 6 simultaneously. I either get a 1 or I get a 6. So

these are exclusive events. And the probability that I get either a 1 or 6 is just the sum of these two probabilities.

So those are the – and following on from that, if we have a x is a random variable, so that's something like what happens when we, like the number we get when we throw a die, then we define a thing called the 'expectation of x' to be the sum of the probability of the ith outcome times the value that x takes on the ith outcome.

And it sort of, roughly speaking, it's often called 'the average of x' but that is to say, if you make a number n of trials and work out the average value that you get of x, you're hoping to get a value – you should get a value which is close to this. It will never really agree with this. But the idea is that as you do more and more experiments, the average of all those experiments will converge – will rattle – in a narrower and narrower range around this expectation value.

Then we have a few simple rules. That if we have two random variables and add their results, and then take the expectation value, then that is the expectation value of x plus the expectation value of y. That's always the case. Whether the events - the variables are independent or not.

So and zillions of branches of science use probabilities right? It's a major feature in medicine, major feature in the financial markets. And they use probabilities in just the same way that physicists do. But physicists have a unique way of calculating probabilities which nobody else uses. And I think this is a central mystery. And that's because, in quantum mechanics we calculate these probabilities through amplitudes.

That's to say, every probability that we're interested in, P is the mod square of some complex number – its amplitude. Its probability amplitude.

So we never calculate this directly. We always calculate a probability amplitude and, having got it, we take it – which is a complex number – and we interpret the mod square of that complex number as the probability. And so all of quantum – the purpose – my purpose in the next few lectures is to persuade you that all of quantum mechanics and all its strangeness follows from this business here. Which nobody else uses.

There is no other branch of knowledge – you know, there are people in the city who talk about the quantum mechanical. There are even people who name their hedge funds 'quantum', etc. They like to have a connection with quantum mechanics but it's completely bogus. Because they never calculate probabilities in this way.

Okay, now the consequence of this is that the probability of – supposing something could happen by two routes, right? So let's be specific. Let's suppose that we have an electron gun. And we have a double-slip arrangement.

Whoops. My drawings look never very good. Something like this.

And we're firing electrons out of here, sort of in a scatter pattern and some of them go through holes and then hit our detector – our screen over here, scintillator, photographic plate, whatever you want to use. And others bounce and then so.

We'll call this S and we'll call this T. There are two – if we focus on a particular place, x, here on the screen, there are two ways in which an electron can arrive there. It can go through the

top hole or the bottom hole. And we'll call the path through the top hole the path S and the path through the bottom hole, the path T.

So if you – what we're interested in calculating is the probability that we get an electron arriving at x. So the probability of arriving at x should be calculated from some amplitude. And the rule is that that amplitude is the amplitude to take the path S plus the amplitude to take the path T. And then of course, that gives us the amplitude to arrive there, regardless right – so this is like the probability rule up there. PA or B – this is the probability that it got there by either route S or route T – is the sum – well up there, it's the sum of two probabilities.

But the rule here is that it's the sum – the amplitude for it is the sum of the amplitudes. And the probability is the square of this. And what does that give us? That gives us – because we know how to take the mod square of two complex – the sum of two complex numbers. This is AS mod squared plus AT mod squared plus AS AT complex conjugate plus AS complex conjugate. AT. So that stuff follows just from the ordinary rules for taking the complex – the amplitude of the sum of two complex numbers.

But this, we know, is the probability that it got there through S. So that's P, that it took route S plus P that it took route T plus this stuff, which can be – this stuff here can be written as twice the real part of AS `[[A star T 0:15:45]]`.

So the probability that something happens when it can happen in two mutually exclusive ways, because it either goes through the top hole or it goes through the bottom hole, is the probability that it – is the sum of the probabilities that it took either route plus this funny stuff down here. That's a consequence of calculating probabilities using amplitudes and this fundamental principle – that if something can happen by this way or by that way, then you add the amplitudes. You don't add the probabilities.

Nobody knows why that's the right rule. You should reasonably ask me, "So how do I know that's the right rule?" And the answer – I think the proper answer to that question is, that this is the fundamental cornerstone of quantum mechanics and our civilisation quite simply depends on quantum mechanics because we're all busy communicating with each other using electronics that has been designed using quantum mechanics.

So it's – of course there are particular, specific experiments that one could talk about. But really, it's not as persuasive as the point that – without this, quantum mechanics would make no sense and without quantum mechanics, our civilisation would fall apart.

Now, let, yeah okay so – let's think a little bit more about this. What do we think that these individual probability distributions look like? In other words if you covered up one of these things and were just firing your bullets through one hole, what would you imagine?

Well of course, your electrons – your bullets – your particles – through one hole, what you would imagine was that the majority –hat the probability would be largest on the place which was formed by a straight line from the centre of the muzzle of the gun, through the hole to the screen.

So you would expect that PS looked like – so I want to draw of a plot of – this is going to be x. I guess I better put this is x is 0. I would expect that PS looks something like this. Some kind of vaguely Gaussian. You know so it's most likely to arrive at – this is the point which is the geometrical – is the intersection of the straight line through the middle of the muzzle and the

middle of the hole, right? And there's some width because the slit has some width, the muzzle has some width and doesn't fire bullets exactly in one direction but in some spray of directions. And we would expect that P of T correspondingly was the same thing on the other side of the origin. Right?

So if these Gaussians are very narrow, we're expecting that P of x, at some location here, say. If we chose this place, we'd find that P of T was about equal to zero. P of S, PS was sort of some number here. So this vanished – this amplitude would vanish because this is the mod square of whatever complex number it is that sits underneath. And so this term would disappear and we would find – guess what? Surprise, surprise that the probability of arriving at x was indeed equal to the probability of arriving through S.

But suppose these things – and now we're interested in the more interesting case where these are really broad distributions. And this is a really broad distribution. Okay? Then there will be places where there's a non-negligible amplitude coming from both sides. And in fact by symmetry, it's evident that at the origin, in the geometrical middle of the screen, there will be equal amplitudes coming from the equal probabilities expected from both sides.

So, in this neighbourhood, we're expecting that this number is about equal to this number. And these two numbers have comparable magnitudes. So let's in fact write A of S is equal to mod A. Let me put a subscript on it. Mod AS e to the i phi S. Okay, so this is a complex number. That's the funny quantum mechanical thing. So it has an amplitude in the technical sense of this is a quantum amplitude. But it has an amplitude in the sense of complex numbers and modulus, sitting in the front here. And then it must have some phase up there.

Similarly, we'll write that AT is equal to AT e to the i phi T. And both and everything here will be a function of position down on the screen. Right? This amplitude depends on where you are on the screen. This does and we expect that this does. We expect all of these bits of the complex number depend on position.

But when we're in the middle here, so near centre of screen, we're expecting that the modulus of AS is about equal to the modulus of AT. Because this is the square root of the probability of getting there and this is the square root of the probability of getting there through S only, this through T only. And we can't see any difference between the two.

So what does the combined probability look like then? P of x is on the order of – it's about equal to – 2 times the probability of getting through S. Because PT's about equal to PS. But now we're going to have plus twice, we're going to have AS mod squared – no, no. Right, because we're saying that up there where you've got A of S times [[A*T 0:22:37]]. But we're saying that the modulus of AS is about equal to the modulus of AT. So I can just put in a – that product just becomes this times e to the i – sorry – times the real part of e to the i phi S minus phi T.

But this we recognise as the probability PS. So this is about equal to 2PS 1 plus – and the real part of this, of this, is the cosine of phi S minus phi T.

So these are what we're expecting. Only, it's only an approximate relation and it's only valid near the middle. But the conclusion of this – the implication of this rule for adding, for adding the amplitudes is that the probability as a function of position near the centre, is going to be what you would naively expect. So this is the classical result. Right?

The classical result is the probability of arriving there is twice the probability of getting

http://rss.oucs.ox.ac.uk/mpls/quantum_mechanics-audio/rss20.xml

there through either one of the slits because each slit's contributing the same probability. But this is now being multiplied by 1 plus cosine of this totally quantum mechanical bit. And this bit is called the quantum interference term.

And the extraordinary – so the prediction is, since this cosine – so this difference – we'll calculate what this difference between the phases is later on. We can't put a number on it at the moment. But we do expect phi S and phi T to be functions of position. And so, by default, we have to expect that this thing is varying with position. And as the cosine – as the argument of the cosine varies with position though, you know, goes 0 and 2 pi and so – cosine is going to go from 1 to -1 and this probability of arrival is going to go from nothing to four times the classical – sorry – to twice the classical probability. Four times PS.

So what we're expecting is that at the end of the day, P of x is going to do some kind of oscillation. This is only valid in a small region of x. But it is an unexpected – it is surely a surprising result. So this is 2 times classical probability. And this is zero.

So that's phenomenon of quantum interference is an inevitable consequence of this extraordinary rule for adding amplitudes and calculating probabilities from the sum of the amplitudes rather than adding the probabilities. That is what makes quantum mechanics special. And that is a phenomenon which doesn't see – nobody else who uses probability encounters the need to do this. Only physicists encounter this need. That I think is the real mystery.

How are we doing?

Okay, of course we have to ask, "Why is it that this – "

If you fire machinegun bullets through slits and stuff, we're not expecting to find that there's a safe place to stand every yard or every millimetre or any distance. These places where no machinegun bullets are doing to arrive, we don't believe exist. And you have to ask the question, "Why not?" And the answer – we will calculate the answer later on. But the answer is going to be that as the mass of the particles you're firing goes up from mass of an electron up to the mass of a bullet, the pattern – this pattern – stays the same but it gets more and more and more and more compressed.

In other words, this distance between places where it's safe to stand gets smaller and smaller and smaller and smaller until it becomes ludicrously small in the case of machinegun bullets. And when you make any measurement with machinegun bullets, you inevitably average over the places where the bullets are extremely likely to arise – twice as likely to arise as in classical physics – and the places where it's safe to stand.

So you inevitably average over these places and you end up with this average. You're unable to measure anything but this average. Nobody has figured out a way to measure anything but this average in the case of things like machinegun bullets. So that's how we recover classical physics. For quantum mechanics is asserting that there really are these places where it is safe to stand.

If you were small enough.

Okay. So now let's have a slightly – let's talk about quantum states.

So my claim is that essentially everything follows from what we've already covered. With

its all a consequence of this interference business through using probability amplitudes instead of probabilities.

But now we have to have some apparatus.

So we have got some – we have in our lab some system, some thing, that we're trying to investigate. So in the simplest case, it would be a particle – a spinless particle. And let's fanaticise about spinless particles. So that's particles which do not have any – that aren't gyros. And let's fanaticise about them although it turns out that spinless particles are very rare. And things like electrons and neutrons and protons even are little gyros.

So if we had a spinless particle, we could – it's a system – it's a dynamical system – and you can ask yourself, "So how do I characterise the state of this particle?" Well, there are things used to characterise its state. Of course by measuring something. And what can you measure? You can measure the x, y and z co-ordinates. You can measure the Px, Py and Pz momentum. You could measure its energy. You could measure its angle momentum. These are all things that you can measure. So there's a range of things that you could measure. And in quantum mechanics these measure – these things you could measure are all called 'observables"

Then you characterise the system by saying what results you would get if you made these measurements. Now in quantum mechanics, remember that we've accepted that there's a probabilistic aspect. So we don't expect to be able to say that if I measure x, I will get the value 3.1415963, whatever, right, metres. I expect to have to come clean and say, "Well, I don't know. There's a probability distribution. I think it's about round here." That's just how life is going to be.

So what do you do? What you do, of course, is you specify the quantum amplitudes to obtain certain results of measurements.

So we characterise the system – the state of our system – by measuring – by giving quantum amplitudes to possible outcomes of measurements. Outcomes. I think that's pretty reasonable.

And it turns out in quantum mechanics that the possible outcomes are sometimes, but not always, restricted. So, if you have an electron which is free to wander the universe, then the possible outcomes of its x co-ordinate can be values from minus infinity to plus infinity. All real numbers are on.

And the range of possible values are what are called the spectrum. So the possible outcomes – the numbers you can get – they form the spectrum. Spectrum observable. So the spectrum of x is generally minus infinity of infinity which is not a very interesting – I mean, so there's no interesting restriction there.

Similarly, the spectrum of Px – the momentum in the x direction – is usually the same. But the spectrum, for example, of the z component of angular momentum – Jz – turns out to be only discrete values. It turns out that we'll show that this is the case that you can have numbers like dot, dot, dot comma, k minus 1 h bar k h bar k plus 1 h bar k plus 2 h bar and so on, where k is equal to either – for a particular particle, it's either equal to nought or it's equal to a half.

So the spectrum can be a discrete set of numbers or it can be a continuous set of numbers. This is a property of the observable. The spectrum of the energy is often a discrete set of numbers – not always – e0, e1, e2 – that you have to calculate by hard grind. And we'll spend a great deal of time calculating the spectrum of h. It's a very – it turns out to be a key to find out what that is

for a particular system.

So all these observables have spectra and how you would characterise the state of the system if you were talking about its energy, is you would give the amplitude. So we could give – we could possibly specify the state of our system by giving the amplitude to get e1 – sorry e0 – the lowest energy. The amplitude to get the next energy above, the amplitude, etc.

So let's call these – let's call this a0, a1, etc.

So if you – the idea here is that for some systems, if you know the complex number – a0 whose mod square gives you the probability of – if you would measure the energy that you got the possible value e0 – and you also knew this number, a1, whose mod square is this probability. And if you knew this number whose mod square was the probability of getting the nth energy level and so on, right? In general they'll be an infinite number of these. If you knew all of these amplitudes, you would completely know – you would have completely specified the dynamically state of that system.

What do I mean by that? What I mean by that is, if I knew all those amplitudes, I could calculate the amplitude to find any other amplitude that you might enquire about. For example, I could find the amplitude to find my system at the place x. Or I could calculate –from those amplitudes, I could calculate the amplitude to find that the momentum is the value P.

So we have the concept here of a set of amplitudes. It's clear – I hope it's clear – that you will need a set of amplitudes to define the state of a system in quantum mechanics.

In classical mechanics, what do you need to know? You need to know for a particle, you need to know x and P. x, y, and z. And Px, Py and Pz. Because then you've pinned down where the thing is and how fast it's moving. And when you know that, you're all done. Six numbers. Done. Because from that, you could calculate the energy, you could calculate the angle of momentum. You know. Done.

But in quantum mechanics it's not – life isn't going to be so simple because we've agreed that you probably don't know what x is and you probably don't know what Px is. The best you can hope to know is what these probability distributions are. And we've agreed that these probability distributions are, for reasons that nobody understands, going to be defined in terms of these complex numbers – the quantum amplitudes whose mod square give the probabilities.

So knowing – specifying a complete set of information is a matter of writing down a long list, unfortunately, of quantum amplitudes. The good news is that you don't need to know all possible – you don't need to write down all possible amplitudes – quantum amplitudes – because there are rules, which we're going to develop, for calculating from a complete set of quantum amplitudes all other quantum amplitudes that might be of interest. And we'll do a concrete example probably next time.

Maybe we already – maybe we do have time to just do this. Yeah, okay. So let's have a look at this.

So, I said that electrons and protons and neutrons and quarks – a huge number of elementary particles – have, are gyros. So they have an intrinsic spin. They are gyroscopes. They have an intrinsic spin. And they're called 'spin-1/2 particles' for reasons that will become apparent in a moment.

Let's just use this. We'll develop the theory of this properly next term. But I want to use this as an example of a complete set of amplitudes and what it enables you to do.

Okay. So, the total angle of momentum of these particles is always the same. They spin at a certain rate so that they have angle of momentum that's root $\frac{3}{4}$ of h bar, where h bar is Planck's constant over 2. So that's the amount of – that's the amount of spin they have. And they just have that spin. And you can never change it. It's always the same. But what does happen is that this – the direction that this angle of momentum points in changes.

So, whereas the total angle of momentum is this, the angle of momentum in some particular direction, for example, the z direction, if you measure it, it turns out that you can only get two answers. Plus or minus a $\frac{1}{2}$ of h bar.

So, and moreover, there is an amplitude, a plus is the – so let this be the amplitude to measure Jz equals a plus a $\frac{1}{2}$ h bar. And obviously a minus will the amplitude to measure that Jz is minus a $\frac{1}{2}$ h bar.

Now in ordinary talk, what we say is – what everybody says. And I will – you'll find me saying this but it's immoral. I shouldn't. Is that if Jz is plus a $\frac{1}{2}$ h bar, its spin is point upwards and you imagine it to be a little particle doing upwards. And when Jz is minus a $\frac{1}{2}$ h bar, you say it's pointing downwards.

And this is a fundamental mistake because, if you square a $\frac{1}{2}$ h bar, you – and take the square root – you don't get that. Right, this 3 indicates that actually this particle has a $\frac{1}{4}$ of h bar – sorry it has a $\frac{1}{2}$ h bar associated with the x and y directions as well.

So it's actually not a good idea to think of it as spin up as being – having the spin pointing upwards. The most that we can say is that really it's pointing sort of not downwards. It's pointing vaguely up and this one is pointing vaguely down. I don't really know which way it is in the x-y plane.

So that's just a little, a little word of caution. People get themselves into a real tangle by imagining that this means the spin is up and that means the spin is down. We all say that and you'll find me saying that but just – when you find yourself saying that, just have a little trip in the brain which says, "But hang on a moment. I mustn't take that too literally because it does have angle of momentum in the x and the y directions. Even though I've measured Jz and found it up and Jz, or Jz and I found it down.

Okay, so the good news is that the set A plus comma A minus is a complete set of amplitudes.

What do I mean by that? What that means is, if I know those two complex numbers – I know both of those two complex numbers – I can calculate the amplitude, and therefore the probability, to find the particle with its spin in any direction that I want, that you specify is either plus a $\frac{1}{2}$ h bar in that direction or minus a $\frac{1}{2}$ h bar in that direction.

And we'll work that out in some detail. Yeah. So, so from these we will – maybe I want to write the formula down. I'm not sure. [[?? 0:42:11]] notes. No, I don't think we do yet. We're not ready to write that down. We just want to make that statement that –