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Contributor Okay. Good morning everybody. Let's begin. So yesterday we looked at this very artificial problem of a particle trapped in a square potential well. And the motivation that I've billed as a reason for discussing these problems was because `[[?? 0:00:21]]` these potential wells with sharp edges was it enabled you to solve some simple problems, which illustrated features of quantum mechanics which are general, which would survive to either more realistic potentials.

But the amount of physics you can do with a single square well is rather limited. There are some physics you can do with it. And I would urge you to play with this to do things like, supposing that your state is a linear combination of two adjacent stationary states so that there is some motion. See that the particle moves to and fro across the well at about the speed with about the period you would expect. The energies in that square well are proportional to the square. They go like n squared of some integer whereas the energies in the harmonic oscillator potential well go like n.

That's to say when we took this simple case of the infinitely deep potential the energy levels go like n squared not like n. So that gives you a very different behaviour from the simple harmonic oscillator and I think it's interesting to investigate that and to find recover classical results. And, see how that dependence of the energy levels on n squared rather than on n manifests itself in the bottom line.

But we won't take time to do that. The lecture course is short. We are going to move onto this problem which is more fun and will allow us to understand how an ammonia maser works which is a time keeping device and was the first masing lasing device. So what we do here is we imagine, so first of all we're going to solve an abstract problem and then I'll explain why this is relevant for ammonia.

Imagine we've got two potential wells which are divided by some barrier. Ideally these potential wells would only be a finite depth so the walls here would cruise off like this with v0. But the computations are made much easier if we let these walls go off to infinity. So the potential goes off to infinity if you go to left of x is minus b or right is of x is plus b. The potential is zero here the potential is v zero some number there. So this is basically two of these square wells put adjacent to each other with a finite, so there's only a finite barrier between the two. And nothing essential has changed just the computations are made a bit easier by making these go off to infinity left and right.

So what are we going to do? We want to find again the stationary states; the states of well defined energy of this system because they will have an interesting property which lead to the ammonia maser. So again here is the origin, x is zero. This potential well is symmetrical on reflection around the origin so it's an even function the potential is an even function of x. That guarantees that the parity operator commutes with the potential energy operator which guarantees that we can look for states of well defined parity. So the stationary states could be eigenfunctions of p so they have well defined parity.

The usual argument about commuting operators mean we can find a complete set of mutual eigenstates. And we're going to assume, so there will be states which have even parity. There will be states that have odd parity and as yesterday the equation we have to solve is minus hbar squared over 2m d2u by the x squared plus v of x u is equal to eu. So we have to solve this equation and we're looking for even functions and we're looking for odd solutions. We are going to assume that we are going to look for states which are low enough in energy that they are as it were bound by either this well or by this well. As I say they're classically forbidden in this region.

So we're going to look for states which have less energy with e being less than this barrier height. So classically the particle will be stuck in one well or stuck in the other well and could go between the two. We'll see quantum mechanically it will go between the two. So that being so the solutions, so what does that mean? That means that we've got d2u by dx squared. Well d2u by dx squared is going to be 2m over hbar squared v0 minus e times u. And this quantity is going to be positive when we're in the middle between x's when mod x is less than a.

So this quantity is going to be greater than zero and that means that in this middle zone the solutions to this equation are going to be things like e to the plus big Kx. So this means for mod x less than a we're going to have u goes like e to the plus or minus big Kx, this is what we wrote down yesterday, where big K is the square root of 2mv0 minus e over hbar squared, okay? Or, because we're looking for solutions of well defined parity and these objects don't have well defined parity we can take linear combinations of those.

And we can say or, looking for well defined parity, we can say u goes like cosh kx arsinh big Kx. So these are the solutions of well defined parity to that very boring differential equation given that this quantity here is positive. So those are the solutions. We know that our solutions will have that form in that middle section as drawn up there. This is x equals a, this is x equals minus a.

And then in the allowed regions b will be somewhere over here, right? In the allowed regions either side of the central barrier we know that we will have sinusoidal behaviour there we'll be trying to solve the equation. So this for mod x less than a. If mod x is bigger than a and less than b we will be trying to solve d2u by the x squared plus nothing times plus v being nothing in that zone times u equals 2m over hbar squared e times u. So we're going to have solutions like sine kx plus some phase constant where k is going to be the square root of 2me on hbar squared.

And this is, so I put in, if I put in a phase – an unknown phase, this is an unknown phase here then this sinusoid will represent any linear combination of sine and cosine. And so will be a general solution to this, oops, sorry. No that's correct. I want e to be negative right? Sorry, no, no, no, no, e is positive, e is positive, e is positive. Yes there's a minus sign here coming from the minus hbar squared over 2m so that's why there's a minus sign here and that's why we have this sinusoidal behaviour.

Right, so we know what the solutions look like in the middle. We know what the solutions look like on the sides. And all we have to do now to fix everything up is make sure the boundary conditions are satisfied where they join. So we say that at x equals a, we have to make sure as yesterday that our wavefunction is continuous and has a continuous derivative. So the continuity of the wavefunction let's specialise on the even parity case. Continuity of the wave function is going to say that cosh x ka must equal b, that's my constant that multiplies the sine, times sine ka plus phi, that's the continuity of u.

And then I have to deal with continuity of the gradient of u as yesterday so we have k arsinh ka is equal to kb cos ka plus phi. Yes. So we are not really very interested in b, it's the same sort of setup as yesterday. We're not very interested in the constant b. Yesterday we had a similar constant a. We're very interested in the value that k takes and this thing here, big K, as yesterday can be expressed as a function of little k. So the name of the game is to get rid of b.

So we divide this equation by this equation. So 1 over 2 leads us to the conclusion that the hyperbolic cof of ka is equal to k over k times the sorry, sorry, sorry, sorry. I'm doing which division, sorry I've got that the wrong way up. So that becomes big K over little K right, because I am taking this equation and dividing it by this equation. So this k was on the bottom as it were and I've brought it up to the top of the other side times the tangent of ka plus phi.

Now we have our, no we haven't quite yet because there's one thing we haven't fixed which is we need to find out about this phi thing. And what do we have to do with the phi? Phi has to be chosen so that the wavefunction in that right hand zone vanishes at x equals b. Remember when we had the infinitely steep sides we concluded that the wavefunction vanished adjacent to the rise to infinite potential energy. That was the point we finished on yesterday.

So what we can say is that at x equals b we require because the potential is about to go infinite, that u equals zero. So that means that sine kb plus phi is zero and kb plus phi is, so what does that imply? It implies that kb plus phi is some integer number of phi's. So that tells us what phi is. It tells us that phi obviously is rpi minus b. So we put this information into here and we have our equation and while we're doing it it's probably a good idea to express big K in terms of little k. So we have expressions for big K and little k on the board and let's work out exactly what we have actually.

So big K is the square root of 2mv zero minus e on hbar squared, which we write as 2mv0 a squared over a squared hbar squared minus 2me over hbar squared is in fact little k squared. And yesterday we identified this object as a dimensionless constant w. So this thing is the square root of w over a squared minus k squared. And so when I take this equation here and that result there in the right hand side and this result here to get rid of those big K's. We discovered that the hyperbolic cotangent of, I want big Ka which is going to be the square root of w minus ka squared is equal to on the right hand side big K over little k, which is going to be the square root of k, sorry square root of what am I talking about, of big W over ka squared minus 1 times the tangent of rpi minus k e minus a.

Now we have what this is, so w is the dimensionless constant that characterises the potential wells on each side as yesterday. K is the thing of interest to us because it controls the energy. It determines the energy of the stationary state and is the only unknown in this equation. Everything else is known. So what we have here is a pretty ghastly equation whose roots determine the values of k which in turn determines the value of energy. And as yesterday the way to solve this is graphically, to plot the left hand side of the equation and the right hand side of the equation in separate curves and see where they intersect.

And I live in hope that my computer has this here. So, oh, the computer may have it here but it didn't come up. Oh dear, the system has perhaps gone to sleep. Is anything showing? Yeah? Not oh but meanwhile my computer has gone and buried it's...hopefully here we go. Right. So that should show, it's very faint from here it's warming up right, but that should show the two sides of the equation. The right hand side of the equation, sorry I am now concerned that this is not, right that was that problem. Right we need to go further down. It's not the right solution. Here we go.

So here is the double potential well and each of these curves is the right hand side of the equation. It's coming down like this and there's a different one because we have this tangent. So the right hand side of the equation contains a tangent which goes to zero. We can forget about rpi as a matter of fact right, because the tangent of rpi plus an angle is the same as the tangent of the angle. So I could write this, I probably should write this as w over ka squared minus 1 times the tangent of this angle here k b minus a. And there's a minus sine out front here which is that minus sine.

So you can add an rpi to a tangent and you don't affect its value. So this tangent keeps having zeros, right? As k increases this tangent hits pi, 2pi, 3pi etc. The argument of this tangent hits pi, 2pi, 3pi etc and the tangent vanishes. And those give you the points where these vertical lines crash down through the origin, these points, these points, these points. That's why there are many branches. The other thing that's happening here so the dotted line there is, sorry there are two, yeah, focus for the moment only on the top curve because, so there are two curves...where is this?

I think this thing needs a new battery. There are two curves running horizontally, almost horizontally, a dotted one and a dashed one. The dotted one is the one we should look at, the one that curves upwards at the end. That is a plot of this cof alright? So when if w is fairly large and ka is fairly small then we're looking at the cof of a large number and that's always 1. So this thing runs along at 1 and eventually when ka becomes on the order of w we're looking at the cof of something smaller than a large number. And that then goes up towards infinity. That then goes up towards yep.

So that's what's happening there and the magic values of k are given by the intersection of the dotted line and the full lines coming down. And you can see you get a series of you know, a series of energies. There are only a finite number of them because when ka becomes bigger than w, when ka becomes bigger than w this square root goes imaginary and we no longer have any solutions to the equation and bad things happen here too. So there are only a finite number of energy levels if the well has only finite value of w.

If we would repeat all this, so that's what we have here we have a finite number of energy levels. This is for the even parity case. If we were looking for the odd parity solutions. So for odd parity what would happen is that only the only thing that would happen is that this would become arsinh and that would become a cosh. Everything would be the same. So the only thing that would happen as we followed through this logic would be that this cof would become a tanh, a hyperbolic tangent.

So for this is even parity, you can follow through the algebra afterwards but believe me I think it's very plausible that all that happens is because you're swapping over a cosh and arsinh this becomes odd we get tanh of this square root equals minus the square root etc. So the vertically crashing down lines remain the relevant lines, the right hand side hasn't changed. But the left hand side has changed. So it starts off as the fangent of the hyperbolic tangent of a large number i.e. 1 and then it becomes the tangent of a smaller number as ka becomes comparable to w. So it turns downwards not upwards because right.

So the point now, the crucial physical point is, the crucial point has physical consequence is that when ka is small, in other words when and k is a measure of the energy right? The energy is proportional to k squared because hbar k is the momentum of the particle as it rattles around inside this well. So the low energy solutions are associated with, these things come in pairs. The energies come in pairs agreed? Because we have the even and odd parity things intersect at low k, almost exactly in the same place. On that diagram you can't see the distinction.

As you get to higher values of k and the particle is only marginally bound, has an energy which is comparable to v0 the two curves for the right and the left sides diverge and you can see you get different values of k. So what the key thing is, the solutions come in pairs. Because we had two potential wells we get two adjacent solutions. In fact if we had three potential wells we would get three. If we have an infinite number we would have an infinite number and in solid state physics this is crucial because in a crystal you have you know 10 to the 24.

You have some vast number of potential wells one for each atom and you get a vast number of solutions all crowded together. Anyway, we just have two wells. We have a pair of solutions. They come in pairs of similar energy, very similar energy. So 1 is even parity and this actually will be the lower energy solution. It has the smaller value of k, you can check from the diagram up there and 1 for the odd parity and this is the higher energy. So the lowest energy, so for this system for a particle trap like that in a well what do we have?

We have a lowest state and just above it which is of even parity. So the ground state, here we are we've got a picture here I think. The ground state so the top diagram there sort of shows these wavefunctions. We have a sinusoid on the right a sinusoid on the left, the ground state is two upper full curves. Its two sinusoids with a depression in the middle where the barrier is and the particles classically forbidden. And then at an energy just a tiny bit higher you have a very similar pair of sinusoids. They are in fact subtly different values of k so they're very, very subtly different but essentially to a very good approximation.

The parity state is the same as the even parity state except reflected around the origin so that it becomes negative on this side. So we have two states. We have so the wavefunctions are going to be well we'll call them u plus of x which is the even parity case and u minus of x, sorry, sorry, u e of x and u odd of x which is the odd parity case. If we take linear combinations of these two states we get two other states psi plus which will say 1 over root 2 of x which is u even of x plus u odd of x and a psi minus of x.

So if we take, so a psi plus is the sum of those two wave functions up there so essentially it vanishes on the left and has a non zero value on the right. So this is state of being on the right and this is the state of being on the left because if you take them away they add up on the left and they subtract on the right. If we put a particle, if we actually put the particle into the right hand well we will set our system up in this state of psi plus. This state of psi plus is not a stationary state. It is not an energy eigenstate because it's a linear combination of two eigenstates of different energy.

So if we drop it on the right our initial condition at t=0 this is what our wavefunction looks like. What does that wavefunction look like generally? Of psi of x and t is going to be 1 over root 2 times u even of x e to the minus i e even t over hbar, the usual boring time evolution of a stationary state plus u odd of x e to the minus i e odd t over hbar. Which we can write more conveniently as e to the minus i e even t over hbar over root 2 even of x plus u odd of x e to the minus i e odd minus e even t on hbar.

And the crucial thing is that if we wait long enough, if we wait such a time that the argument of this exponential becomes equal to pi, we'll be looking at e to the i pi which is minus 1 here. And our state will have evolved into some phase factor who cares times u even minus u odd which is the state of being on the left. So after a time the time required for this to become pi which is t=pi hbar over e odd minus e even the particles on the left. And you can see that this will go on forever.

You put the particle in on the right it's not a stationary state. After this time it moves to the left and then after twice this time it will have come back to the right. It's going to oscillate between these two wells forever and ever according to this theory. And crucially the timescale is long. It takes a long time to get from the right to the left if this energy difference is small and we've seen that this energy difference is small, right? So this energy difference being small means that this timescale is long. Where is this energy difference small? It's small when the barrier between the two wells is high.

So what we say is that the particle takes a certain time to tunnel through that barrier. So we say that the particle tunnels through the barrier in a time which grows, it grows very rapidly as a fact with v0 with w with the height of the barrier. So a high barrier or a wide barrier means it takes a long time to get through but according to this now it says it will eventually get through. Okay so that's just a toy problem. Now let's see what the hell that's got to do with the real world by talking about ammonia.

Somewhere here we have a picture of ammonia. So ammonia is a, we don't have a picture of ammonia, no sorry, I've somehow left it out. Okay. So what does ammonia consist of? It consists of three hydrogen atoms so these are h's and it consists of a nitrogen atom and the three hydrogens form some kind of a triangle roughly speaking and the nitrogen atom sits, well this is the classical picture. We will see quantum mechanical picture isn't like this. But this is the classical picture. We think of the nitrogen atom as sitting either above the triangle formed by the hydrogen atoms or below the triangle formed by the hydrogen atoms.

Now this is a complicated system. It's got four nucleii and ten electrons and so it's a really complicated dynamical problem. But next term in the next course you will discuss a thing called or see a thing called the adiabatic approximation which is what chemists use in order to understand the dynamics of complicated systems like this. It allows you to treat all these bonds between the nucleii which are provided by the electrons as springs. So what we can do, what chemists routinely do do is they calculate the energy of this molecule in what's called a clamped nucleii approximation.

So they say, "Let the nucleii, you know, let them be here and let's calculate the energy." And you can do that for the nitrogen nucleus being at every point along the line that passes through the centroid of the triangle right? So imagine doing that. That leads to a potential energy curve if you like of the hydrogen nucleus which is going to be something, it turns out sorry it's not obvious that it's going to be but it turns out to be something like this. So there are two wells. The lowest potential energy when it's an appropriate distance either above the plane or below the plane.

There are two wells and this sort of barrier in the middle because it isn't comfortable being in the middle of the triangle formed by the hydrogens. So we have this kind of potential energy curve and now we know what the energy levels of this molecule are going to look like because we know that there are going to be even parities the stationary states are going to divide this, it's obvious on geometrical grounds this is symmetrical. That this well looks just like this well because there's no fundamental difference between above the triangle and below the triangle.

So we know the solutions are going to come in pairs. There is going to be, of even parity solutions and odd parity solutions. Just like in our square well. So we also know that if we start, if we start with a nitrogen in this well. So we start with a nitrogen say above the triangle it's going on some characteristic time to move over here and then it's going to move back there. And it's going to oscillate between above the triangle and below the triangle. So if we start it in either above the triangle or below the triangle it's going to oscillate between the two.

Those of you who've done chemistry will know also that this nitrogen is going to be carrying a negative charge and these hydrogens some amount of positive charge. So what are we going to have? We're going to have an oscillating dipole This molecule is going to have a dipole element because of the electro negativity of the nitrogen. And it's going to be an oscillating if, if we start it in this state of being above the triangle it's going to oscillate to and fro and what does an oscillating dipole, electric dipole do? It radiates.

So this thing is going to radiate and you will be able to, if you know what this potential surface looks like you'll be able to calculate the frequency of which it's going to radiate because the frequency at which it's going to radiate is going to be given basically by this formula here for the half period to go from there to there right? So twice this time it's going to be the period of complete oscillation and that's going to be the period of the radiation. Okay. So the molecule has started, it's a dipole, so started on top if you can achieve that. It's going to be an oscillating dipole.

And the experiments lead to the conclusion that the frequency `[[nu 0:35:34]]` is about 150 megahertz. You can measure it. So it's a sort of microwave radiation. From this 150 megahertz you can also work out what this energy difference is. So e odd minus ee turns out to be about 10 to the minus 4 electron volts. So that's an energy difference which is small compared to the thermal energies of molecules just here in the room right? At room temperature the thermal energies of molecules - So k the Boltzmann constant times t of the room temperature is something like 0.03 electron volts.

So this is much bigger than e0 minus ee. What does that mean? From your statistical mechanics course you either know now or will soon know that that means we expect at room temperature there are essentially equal numbers of molecules in the even and the odd states. Right? Because the thermal energy, the energy required to go from the ground state – this is the ground state or this is the energy of the ground state to the first excited state is less than the characteristic thermal energy knocking around in the room. So there will be large numbers of molecules in both these even in odd states.

And the idea behind an ammonia maser is to find a way of separating the molecules which are in the excited state, the odd state, leading them into some kind of a resonate cavity where they will then, because they're in the excited state they will decay into the ground state if they're left alone. They will decay into the ground state emitting a photon with a balance of the energy right? So they will radiate away at 150 megahertz. So that's the strategy behind a maser. So to get a maser you have to isolate roughly half of the molecules in the first excited state.

And the way that you do this is you exploit the fact that well let's just imagine putting these molecules into an electric field. So we're going to put the molecules and the strength of this electron field I am going to denote with curly e to distinguish it. So the electric field is distinguished from energy, ordinary e, Roman e Okay? So we put these molecules into an electric field e and ask ourselves, "How does that change? How is that, since these are dipole molecules right, these molecules are electric dipoles, we're expecting that that changes the energy of the molecules putting them in a field. And maybe we can get them separated by exploiting this fact."

So you put them in an electric field and ask, "So how does that change the", we're doing quantum mechanics, so change in energy means change in Hamiltonian. So we have to ask ourselves now, "How does this change the Hamiltonian of these molecules? What term in a Hamiltonian is introduced by this electric field?" And the way to do that is to think in terms of the Hamiltonian, write the Hamiltonian in the basis. So we're going to let plus be, so it's going to be mathematically 1 over root 2 of the even parity, the ground state, plus the odd parity, no, no, using a different notation, even parity plus odd parity.

So this is the ground state, this is the first excited state, this linear combination we're going to call it plus and it will be the state in which the molecule is definitely above the triangle. And similarly minus is going to be 1 over root 2 even parity state minus odd parity state is going to be below. So this is below triangle, this is above. This is exactly what we did over here with our upside plus and upside minus. We were working there with wavefunctions here. We're working with the underlying kets.

The thing is the dipole moment of these two states have opposite sines because this one has the negative charge at positive z and this one has the negative charge at negative z. So they have opposite dipole moments. And let p now be p be the dipole operator, it was the parity operator earlier on but now let it be the dipole operator. Then these states are going to be eigenstates of this dipole operator. They have well defined dipoles. P plus is in fact going to be minus qs times plus where what am I saying?

I am saying this is some charge. This is some distance. The product of the charge and the distance gives me units of dipole, electric dipole moment. I've got a minus sign here because this state I said has a negative charge above a positive z. So the dipole moment points in the opposite direction to the location of the nitrogen atom and therefore this one has a negative dipole moment. And this is eigenvalue, right, is the dipole moment. This is the eigenvalue of this operator because this thing is a state of well defined dipole moment and this is where the dipole moment appears.

And basically this is just some number which has the dimensions of a charge times a distance which you know the charge is going to be on the `[[order 0:42:35]]`. It's going to be some fraction of electron charge and this distance is going to be the characteristic size of the atom. A tenth of a nanometer. Similarly p minus is going to be plus qs minus. So this encapsulates the crucial point that these two states have oppositely sined dipole moments.

What is the energy of a dipole in an e field? Well the answer is that the energy is minus, this is just classical physics, is minus the field times the dipole moment. So now we know how much the energy of this state and this state are changed by the electric field. It's given by this here where we put in those kinds of eigenvalues. So now what we want to do is write down the matrix that represents the Hamiltonian in this plus minus basis.

So we want to write the Hamiltonian as plus h plus a matrix this is a complex number right? Plus h minus minus h plus. So these four complex numbers we want to write down that matrix. And the Hamiltonian consists of, the Hamiltonian of the undisturbed nitrogen molecule plus the contribution here this is plus the contribution here from the electric field. What do we have for the undisturbed, let's calculate this separately.

So I need to calculate what plus h plus is for the isolated atom. Well that is going to be 1 over 2 even, because these are linear combinations of the eigenstates even and odd. Even plus odd h even plus odd right? Because this is this and this is this. But h on, this is an eigenfucntion of this operator if this is the isolated operator. So we get h on e which produces e even times even. Even is orthonormal to odd. But meets up with this so we get an even and an h on odd produces e odd times odd which is orthonormal to this. But meets up with this to produce an e odd. So at the end of the day we get a half of e even plus e odd which is the average energy. Let's call it ebar.

Similarly we have that, let's have a look now at plus h minus, the off diagonal element. That is going to be 1 over 2 even plus odd h even minus odd. So things are rather similar except we now get from here, here and here e even and from here, here and here we get minus e odd. So we get a half of e even minus e odd which I think we're going to call this minus a. So this is a definition of a and we're going to put this minus sign in here because we know that the odd energy is bigger than the even energy. So this quantity here is a negative number and putting that minus sign against the positive number.

So this matrix up here what's it looking like? It's looking like ebar from here sorry a. This is a Hermitian matrix and what appears down here has to be the complex conjugate of that real number a, in other words a and we will find that this one is also ebar. That's for the isolated atom. Then we have to add on to the contribution from this which but this thing is an eigenfucntion of p. So the only contributions down the diagonal, there are no off diagonal contributions from this additional piece to the Hamiltonian. And what I have is plus, this is what I need is the dipole operator.

So I have p on plus which produces minus qs times plus. So this minus sine and the minus sine that I've just spoken cancel and we get plus q curly e and here we get minus q curly es. So this is what the Hamiltonian looks like in this basis. And I'm running out of time so let me just sketch how the thing goes. So what we have got is an explicit expression for the Hamiltonian in the presence of an electric field. What we would like to know is what the energy levels are right, that result from this and how those energy levels vary as a function of the electric field.

But that's a dead synch. What you've got up there is a two by two matrix. Anybody can find the eigenvalues of that two by two matrix. And those will be the energies the molecule has when you stuff it in this electric field. So we find the eigenvalues of this matrix and we plot how they depend on curly e. So what they actually are is the average energy plus or minus the square root, I believe of a squared minus q – just make sure I didn't make a mistake `[[there numbering 0:49:41]]`. Yeah. So these are the possible energies and if we plot this graphically what do we see?

What we see here we have the strength of the electric field. We find that we have solutions that behave like this. Here we have ebar plus a. Here we have ebar minus a. And here we have the strength of the electric field. This is just a graph of those two things. You can easily check that that's the case here of course its ebar. So this is the ground state. If you switch off the electric field we have two states here and here separated by this small amount which is 2a.

As you switch on the electric field the energy of the ground state goes down but the energy of the first excited state goes up. There's a lot of interest in the way which this energy behaves. Here it's behaving quadratically as a function. This is a sort of a parabola as a function of e and eventually it becomes a straight line as a function of e both top and bottom. And what that's telling you is that in this regime here the even state, sorry the ground state which is the even state, it has no dipole moment intrinsically because the particle is equally likely to be above, the nitrogen is equally likely to be above or below the plane.

So in the ground state there is no dipole moment. You switch on the electric field and you make a bias so that the upper fields say, the upper position gets to have a lower energy in the lower position because the electric field is pushing it that way. So the nitrogen starts to spend a bit more time above than below and now the molecule requires a dipole moment, right, because it's not spending equal time above and below. So it requires a dipole moment. The magnitude of this dipole moment is proportional to the energy.

And therefore the energy that you get, the change in the energy which is equal as it says up there e is minus electric field times dipole moment becomes proportional to dipole moment squared. So that's why we have a quadratic dependants like this. Once you're in this regime you're in a strong electric field you've made such a strong bias. You've made being up so energetically more favourable than being down that the nitrogen is spending all its time up.

The dipole moment is now independent of the strength of the electric field because increase in the electric field doesn't cause it to spend any more time up than it's already spending all it's time up. So that the energy becomes proportional to this. How do you make your maser work? You make your maser work. Well the fact that the energy of one of these goes down, the energy, the other one goes up means that as the field increases that if you have an inhomogeneous field, if you pass your beam of nh3 molecules through a region in which the electric field is varying in strength.

So I have some sort capacitor plates, `[[?? 0:52:28]]` plates. I have a pointy thing right, a needle in a cup then the electric field, the field lines go like this and we have a high field up here and a low field down there. So I have a region of inhomogeneous field. The ones in this state are going to be bent, sorry, they're going to be bent upwards. They like the field. It lowers their energy so they move into the region of high field. These ones move into the region of low fields. So this is the ground and this is the excited. And then you can put these ones into your resonant cavity and hear it sing.

So there's an example of how with a very simple minded model you can explore some quite interesting physics and some physics which is very, very inherently quantum mechanical. This energy levels coming in in pairs, because we have even and odd parity states this ability of a molecule which you would think was inherently dipolar in the ground state have no dipole moment because the nitrogen can be simultaneously above and below. All of these features are vary quantum mechanical and with a simple minded model we're able to explore them. Okay.