Title	*017 Einstein-Podolski-Rosen Experiment and Bell's Inequality*
Description	Seventeenth lecture in Professor James Binney's Quantum Mechanics Lecture series given in Hilary Term 2010
Presenter(s)	James Binney
Recording	http://media.podcasts.ox.ac.uk/physics/quantum_mechanics/audio/quantum-mechanics17-medium-audio.mp3
Keywords	physics, quantum mechanics, mathematics, F342, 1
Part of series	*Quantum Mechanics*

Contributor So yesterday I gave a very speedier, I apologise a too speedy introduction to the problem of compound systems but we have a very tight budget of lectures and I wanted not only to talk about, explicitly about these composite systems and how you handle them but also I think it's very valuable to discuss this classical Einstein Podolsky-Rosen experiment because it goes to the core of whether Quantum Mechanics is correct and what it really has to say about the universe.

And there wasn't going to be time and so I could've spent two lectures doing what I did yesterday but it's been compressed in order that we can talk about this experiment which is an important, crucial application of this apparatus of how we apply quantum mechanics to compound systems. So Einstein is famous for saying that "God is sophisticated but he does not play dice." He disliked the probabilistic aspect of quantum mechanics not that he disliked or disapproved of the use of probability in physics. His thesis work had been on kinetic theory and statistical physics so he was quite comfortable with the idea that in classical physics you use statistical methods, probabilistic methods to do things like kinetic theory but he understood that the reason you were doing, you were in that case you were doing probability theory because you had incomplete information. So when you lack information it's obvious that you have to – you have to assign probabilities. The thing that's worrying for him was worrying for him about quantum mechanics was that it asserted that even when you had complete information which we know is embodied in a set of – a complete set of amplitudes still the outcome of experiments is probabilistic and uncertain.

And he felt that this was wrong because and the relation to god there I guess is that an omniscient god would not have the future uncertain, would know what the future was. So god must know something that we don't know. We have an uncertain future because we're short of information but the information must be there. We don't know – we just don't know the information so there must be some variables, some variables which encode the information about what's going to happen. Which if you knew them and at some future time in physics perhaps you would know what these variables were and then you would be able to predict exactly what was going to happen.

And in 1935 Einstein with Podolsky and Rosen proposed this – described this thought experiment which they argued demonstrated that there must be in fact be these sorts of hidden variables. In 1964 I guess it was John Bell analysed a similar experiment and showed that the predictions of quantum mechanics are actually incompatible with the existence of these hidden variables. And then in 1972 20 years after Einstein's death, an experiment of this type was actually conducted and the – and many since have been conducted and these measurements vindicate the predictions of quantum mechanics and therefore prove these hidden variables cannot exist. So that's what the agenda is today to describe this.

So what's the – the experiment that Bell describes is this which is there are various versions of this but the key idea is the same. Suppose you have some nucleus which is going to be unstable and it's going to emit a positron in this direction – well it does emit an electron in this direction. And Alice sits over here and measures the spin, she measures the spin of the component of spin of this electron as it comes by in a direction of her choice we will call it for A for Alice. And Bob sits over here and he measures the spin in the direction of his choice. The component of spin in the direction of his choice which of course we will call B.

So let's imagine Alice acts first. Sorry and the idea is that we know from nuclear physics that both before and after the decay the spin on this nucleus is zero. We know of course that electrons and positrons are giros they carry their spinning particles, they carry angular momentum. And because the angular momentum change of the nucleus is zero it must be that by conservation of angular momentum the spin of this is oppositely directed to the spin of that. So that the angular momenta of the electron plus the positron is zero.

So supposing Alice measures the spin first and Alice gets that A.S turns out to be plus a half. So she finds that the component of spin along her chosen vector A is S. What she then says to herself is this. If Bob measures along A, if Bob chooses to put B equal to A then he's guaranteed to find the answer minus a half right because if he measures with B along B the component of the spin along B – Vector B which is close to the Vector A he's not very likely to get plus a half. He's most likely to get minus a half but I can't guarantee that he'll get minus a half.

Right so the point of that is that that kind of thought that Alice's thought process makes it absolutely clear that Bob's measurements are going to be correlated without Alice's measurements. And what we want to do now is put that on the quantitative basis and ask what does quantum mechanics have to say about this?

Okay so what we want to do is talk about the correlations between the measurements that Bob makes and the measurements that Alice makes given that Bob is going to choose Vectors B at his discretion. So let's talk about the quantum mechanical predictions. So what we do is we choose which we're free to do the Z axis to be along Alice's Vector, we can do that without loss of generality. And we will find – I mean this is – I'm about to write down a result that will emerge in the next couple of weeks from our work in angular momentum – may be work that'll emerge next week from our work in angular momentum but for now I have to ask you to take on trust that because the electron plus the positron together have no angular momentum it must be that their wave function could be written like this as E plus, P minus, minus E minus P plus.

So that is to say that the states – we know that a spinner half particle again this all needs to be justified properly when we do angular momentum but we anticipated these results once before early in the course. That a spinner half particle has a complete set of states which are the state in which you're guaranteed to get plus along some direction for the spin and minus along the direction for some spin. They're a complete set of states. So here are the complete set of states for the electron. Here are the complete set of states of the positron and this says that there is a probability of a half – right this 1 over route 2 is the amplitude to find that the electron is plus in the Z direction and the positron is minus in that direction. And this is – and there's a similar amplitude with a minus sign for the opposite possibilities.

So the origin of this expression will emerge shortly. I must ask you to take that on trust. So this is the state of the system, the composite system of the electron and the positron taken together. So we talked about the collapse of the wave function in these circumstances yesterday, Alice makes her measurement, she finds plus which means that she collapses the wave function into this so this is before Alice makes her measurement. After Alice has made her measurement and found plus we have the upsi is simply E plus P minus, which is to say that there is unit amplitude that Bob will find minus if he measures along the Z axis. In other words if he chooses B to be the Z axis which we've established is – which we've chosen to be the same direction that Alice chose.

So that's consistent with what Alice said. What happens if he takes to be some other direction well what we need to do is express some other directions but is so. We need to write the KET so we would like to calculate the amplitude or the probability that if – that when B uses some other direction he finds it to be positive. That he finds that S of his positron is along that direction. Has plus a half along that direction.

So in order to do that I have to ask you to take something on trust that we will derive but we've seen before which is that this thing is equal to sin theta upon 2, E to the I FI upon 2 of positron down plus cost theta upon 2, E to the minus I, FI on 2 for the positron plus. So what does that say? That says that the state of having of being certain to give you a half along the vector B for the positron is given by this amplitude that's just some complex number times the amplitude – times the state where you will definitely get minus the half on the Z axis plus this amplitude times the state in which you're guaranteed to get plus a half on the Z axis for the positron. And theta and Fi are the polar angles of B, right. B is a unit vector so it is defined by a couple of angles and there theta and Fi they give you the orientation of B with respect to the Z axis. And here we're using the complete set of states along the Z axis. So that's a result that we will derive but I'm asking you take it on trust for now.

So what's the probability that Bob measures plus on B the answer is that we – it's according to the dogma of the theory it's this. Because the state of the system, the state of the positron after Alice has made his measurement, her measurement is definitely this. So this is – that's how it works, the apparatus. So basically we flip this around, we take the Hermitian adjoint of this thing, bang it into minus and guess what we get the complex conjugate of this coming out.

Oops that's sin theta on 2, E to the minus I Fi and I've written on 2 and I've written the probability which means I need to do a mod square – do a mod square, this factor goes away and we're looking at sin square the theta on 2. It follows straight away you could also calculate it that the probability that Bob finds minus on B is 1 minus the probability that he finds plus on B is equal to cost squared theta upon 2. It follows straight away you could also calculate it that the probability that Bob finds minus on B is 1 minus the probability that he finds plus on B is equal to cost squared theta upon 2.

So that puts precisely on a quantitative basis what Alice said. Alice said that if Bob chooses Vector B which is very similar to my A which is the case when theta equals nought then he's guaranteed well if he's identical he's guaranteed to find minus because this becomes 1 and that becomes nought. And if he chooses vector B which is similar to my vector A he's not, it's not guaranteed that he'll get minus but he has only a small probability of getting plus and that's because theta would be small and his probability's looking at sin squared theta upon 2.

So what Einstein-Podolsky and Rosen said – well the question is why is the result that

Bob gets somehow dependent on the measurements that Alice gets. And in particular it looks like the result of Bob's measurements depends on which direction Alice chose because this angle theta is the angle between Bob's vector and Alice's vector and we can imagine that Alice – let's imagine that Alice goes first and chooses a direction apparently Bob's – the probability of Bob's – of outcomes depends on theta therefore depends on Alice's choices. But supposing these positron and electron are sent out at relativistic speed, perfectly plausible that they are then Alice and Bob. Alice makes a measurement and Bob can make a measurement in the rest frame of the nucleus at essentially the same time and if Bob acts that quickly then there is no time for a light signal from Alice to reach Bob setting out after Alice has made her measurement.

So Bob definitely makes his measurement in complete ignorance – has to make his measurements in complete ignorance of what choices Alice may or may not have made and indeed if in this relativelistic case it's easy to see that who acts first, different observers, observers moving at different speeds with respect to the nucleus and Alice and Bob will disagree about who acts first. The whole question of who acts first is neither here nor… It clearly can't affect the physics because it's an observer dependent statement according to relativity.

So how is it that the result of Bob's measurements depend on Alice's choices when it's not logically possibly for a signal to go from here to there in order to effect it. Well Einstein, Podolsky and Rosen said what it must be is that actually the result of Alice's measurement is preordained. We don't know what the result is going to be but that's because we're pig ignorant but god knows. It's for ordained because the result is encoded somehow in the state of the electron not written on the board here because we're using this clapped out quantum mechanical rubbish and similarly inside the positron there's also this magic information, this DNA, this whatever which for ordains as a result of Bob's measurements and then everything is okay.

That was their interpretation of this problem. So now let's talk about Bell's inequality. So that was the state of affairs for I guess 30 years right, 1935 until 1964.

So John Bell said "Okay so let's calculate something. Let the Sigma A be the result of Alice's measurement – whoops. And it's obviously going to be plus or minus a half right whatever she number comes up with is going to plus or – either plus a half or minus a half. And simply and similarly Sigma B being plus or minus a half is a result of Bob's measurement. And let's calculate the expectation value of Sigma A, Sigma B. So there are four cases to consider because they can both measure plus or half, they can both measure minus a half one can measure plus and a half and one with a minus a half and that in two different ways.

So this thing is going to be – there are four possible values that – sorry sigma A times sigma B can be either – could be either plus or minus a quarter. And the possibilities to consider are the probability that Alice gets plus times the probability that Bob gets plus given that Alice gets plus, plus the probability that Alice gets plus and Bob gets – sorry Alice gets minus and Bob gets minus the probability – the probability that Bob gets minus given that Alice has got minus. Right, so in both these cases that product is going to be plus right because in this case this is going to be a plus a half and that's going to be plus a half.

In this case Alice – Sigma A's going to be minus a half and Sigma B's going to be minus a half, the product's going to be plus a half. And then we have some minus cases which is the probability that Alice gets plus say and Bob gets minus given that Alice got plus. And then we have minus the probability that Alice gets minus and Bob gets plus given that Alice got minus. Okay we have to make Bob's probabilities conditional analysis because we've seen that they're correlated.

We can argue that the probability that – for Alice to get plus is the probability for Alice to get minus namely it's a half right we don't – when Alice makes her measurement we don't know a blind thing. So both possibilities are equally likely. So that must be what these probabilities are for Alice and we've just worked out what the probability – we've just worked out what the probability for Bob to get plus was you know we've worked out these probabilities. So we know that the probability for Bob to get plus given that Alice got plus we found that that was sin squared theta on 2, right. So that sin squared theta on 2 by symmetry you could work it out but by symmetry this will be sin squared theta upon 2, right. Because if Alice has got minus we know that Bob is jolly unlikely to get minus if he chooses an angle, a vector which is close to A. And we figured that these, that's this one here we've already shown that the probability for Bob to get minus given that Alice got plus we've already shown is cost squared theta on 2.

So both of these are going to be cost squared theta upon 2 and both of those are going to be sin squared theta on 2 which means that sigma A, Sigma B expectation value is a quarter of sin squared theta on 2 twice over because we get two times like that minus twice cost squared theta upon 2 oh times a half, sorry, sorry, sorry. So here's a half and here's a half from the probabilities of A so those 2s are not really there and one is this – cost squared minus sin squared is cost twice the angle. So this is minus a quarter of cost theta. And what is cost theta, cost theta is actually A.B it's the angle between A and B so this is minus a quarter of A.B. So that's what quantum mechanics predicts is the expectation value of the product of these two measurements.

So now what Bell did was calculate what this would be in a hidden variable theory. So draw a line now we're into another conceptual framework. What we're going to say is that – so there is some function Sigma E which will depend on – so what's this? This is, this thing here is the value that you will find for the spin – the component of spin of the electron along the vector A. We think this is random variable because we don't know the values taken by the hidden – this is a set of hidden variables. This is an N vector with components which are the hidden variables that we don't know but Einstein-Podolsky and Rosen claim just exist to make the outcome of these experiments causal. So this is not a probabilistic quantity this is something – this is either a half or it's minus a half, right. Depending on the values that these variables hidden from us about which we do not know and of course on the direction in which you measure the component of spin. Alright, so this is equal to plus or minus a half in a causal way. And similarly there must be Sigma P this is the positron spin that's also going to be plus or minus a half depending causally on these things. We don't know what this function is, we don't know what these variables are. We don't know how many of these variables there are or anything. But we jolly well do know by conservation of angular momentum that it is minus Sigma electron at V and B because we know that the positron spin is oppositely directed to the electron spin by conservation of angular momentum so if you get plus a half here you were certain to get minus a half here.

So this is – this equality is conservation of angular momentum. So what we do now is evaluate the expectation value which quantum mechanics told us so we do Sigma E depending on A times Sigma P depending on B expectation value we write this out as in classical probability theory. Now what's that going to be well this expectation value means averaged over all possible values of the hidden variables the things that we don't know alright. So the reason that this things seems uncertain to us is because this thing is unknown to us and we therefore we think of this as a random variable. So what's this expectation going to be it's going to be an integral over the components of V we have the sum over all possible values of what we don't know times some probability density that we don't know times Sigma E of V, A times Sigma P of V, B.

So basically we just take an average of this product which is completely determined by V and then – but we take an average of this appropriate way over all the possible values of V to get the experimental expectation value. Standard probability theory. The next thing that we do is we replace this by the corresponding sigma E using that switch of sin business. So we argue that this is the minus the integral D to the N, V rho of sigma E V, A Sigma E V, B. So the only thing that's changed here is we've acquired a minus sign and that P has become an E.

Now we say "Okay now let's imagine that we make this measurement with some other vector, right. Supposing we now calculate the same expectation value between A and the Vector C, just some other Vector. And then we have that the expectation value of Sigma E A, Sigma P, B, minus – that's a complete expectation value. Minus the expectation value of Sigma E A, Sigma P C some other Vector C and what's that going to be according to Zaparatus it's going to be minus the integral D to the N V rho depending on V open a bracket – er, no, Sigma E of V, A will be a common factor and then we will have Sigma E of V, B minus Sigma E of V, C.

Right, because the right hand sides are both going to have this factor because we've taken the expectation value using Sigma E of A, Sigma E of A in both cases. And what will differ in the two cases is that term in the back so one time it'll be B and one time it'll be C so that's what we get. Now `[[?? 0:28:31]]` does something slightly niftily he makes the observation that well – but he knows that Sigma squared V, B is a quarter. Because he knows that this number is either plus or minus a half depending on the values taken by V and B. The square of this number is guaranteed to be a quarter.

So we can say, we can insert into here we can insert a 4 Sigma squared E of V, B without any harm, right. Because we're just inserting a 1. So he says that this expectation value, this commodity I'm not going to write it out again. This expectation value on the left is minus the integral D to the N V rho Sigma E of V, A. I'd better write it out. 4 sigma squared E V, B brackets sigma E V, B minus sigma E of V, C. Very helpful I'm sure.

What we now do is we take, we break this Sigma squared into sigma and sigma and we take one of the sigmas inside here. When one of these sigmas comes in here we get a sigma squared again which is a quarter times 4 is 1. So we get a 1 appearing here and then we and then of course this sigma that I brought in appears there as well so the next line is this is equal to minus D to the N, V rho sigma E of V, A, Sigma E of V, B brackets 1 minus 4 times sigma E of what... We've carried this one in V, B and we've already got one there which is a sigma E of V, C – close brackets, close brackets.

So this is what that expectation value at the top is – it's this. So why's Bell done this. What we now argue is that this bracket so this product of things here is going to be either plus or minus a quarter, right. Because all of these things, they're causal functions and they're either equal to plus a half or they're equal to a minus a half. So this product is equal to either plus a half, a quarter or minus a quarter, we don't know. But whatever happens, so this bracket is either equal to zero or something positive. In fact the brackets are equal to 2 or 0, right. So what we really need is that this bracket is greater than or equal to 0 it's not negative. And this thing in the front here is a fluctuating quantity it's equal to plus or minus a quarter. So what we can argue now is let's take the modulus of both sides, the modulus of the left side is whatever it is, the modulus of the right side just means we drop this and we can argue that this integral, this integral is going to be smaller than the actual integral here is going to be smaller than what we would get if we replace this with plus a quarter because sometimes that is minus a quarter and will be taking away from the integral given that this thing here is never – this thing here is never negative. There's no way that we can never get a positive result – a positive contribution to the integral when this is negative. So if we assume that this is always positive we're going to overestimate this integral.

So let me write that down. We will overestimate integral if we replace Sigma E V, A, Sigma E, V, B by plus a quarter. Because sometimes it's minus a quarter and that minus sign is never cancelled by a minus sign over here. So then I can argue that the modulus of the left side which unfortunately I now have to write out again that's P sorry. Sigma E of A, Sigma P of C the modulus that's an expectation value now I need a modulus sign is less than or equal to because I'm going to write down something which is too large I deliberately made it too big of the integral role. That has been – that factor has been replaced by a quarter, this quarter could be taken outside and then we're staring at 1 minus 4 Sigma E this is of V, B, Sigma E of V, C.

Now we make the observation that the integral this – so we break this integral into two parts it's this stuff times 1 but that integrates up to 1 because this is a probability density and a probability density has to be structured so that if you integrate a probability density times overall, all parameter space you get 1. So this and this make a quarter. So this thing I'm going to write down what it's equal to which is, it's equal to 1, excuse me it's equal to a quarter brackets of 1 from here then now let's consider this on to this stuff here. This on to this stuff here is roughly speaking where we came into this that this times this was the expectation value of sigma on sigma. And this minus sign we can soak up by changing that back into a P. That's retracing logic that we did up there so this becomes 1 plus 4 times the expectation value of sigma E, B, sigma P of C expectation value where the V has disappeared from here because we've done an expectation value operation. We've averaged away all the V dependents in the proper way.

So we have, this is Bell's inequality that we have here now. It's a statement about expectation values associated with the two particles and three possible vectors A, B and C. So the next thing to do is to ask are the predictions we've calculated the predictions of quantum mechanics for these expectation values, we've already done that. So the question to ask now is are the predictions of quantum mechanics consistent with this inequality.

I guess we need to be able to see everything simultaneously and I've not had that right. So let's write the – let's write down here let's find the prediction of quantum mechanics. This is the crucial thing that the prediction of quantum mechanics is that this product which in the other calculation for reasons which if you stare hard at it you realise that there's a notational issue, there's a reason for this. This in the hidden variable calculation is called Sigma E, Sigma P because remember Bob is measuring the positron, Alice is measuring the electron. So this is actually the same physical quantity that we've calculated down there and it's equal to a minus a quarter of A.B. So we can go straight back – so now we put in Sigma EA, Sigma EPB expectation value is minus a quarter A.B which is from quantum mechanics. What does that do? Well let's check out the left hand side what does the left hand side look like it's going to be the modulus of a quarter A.C well minus A.B. Right so the overall minus sign gets lost but it's going to be the modulus of this thing here. That's the left hand side, what's the right hand side going to be it's going to be a quarter of 1 minus A.B, sorry B.C.

So now we need to ask ourselves is it true that this right hand side is bigger than this left hand side. And in this matter we can choose A, B and C exactly as we will, right. Because Bell has shown that any vectors A, B and C his inequality has to hold if there are hidden variables. There's so far no restriction on A, B and C there any three vectors. So and if the quantum mechanical results violate Bell's inequality for any vectors A, B and C then quantum mechanics will be inconsistent with these hidden variables. So at this point we do a choice we choose A.B equals 0 and we choose C is equal to A say cos upsi plus B sin upsi.

So what are we doing? We're simply – upsi is some angle. We're just choosing A and B to be orthogonal vectors and we're choosing C to be a vector between A and B and we've got ourselves a parameter upsi which allows us to move C from pointing along A to pointing along B in a continuous way. So just concretely the picture is here is A, we're choosing A to be this way, we're choosing B to be that way and we're choosing C to be like that. Somewhere in the plain. Stuff it in and what do we get we find that the left hand side is the modulus of a quarter A.C is A.C is cos upsi. A.B is nought. And the right hand side is a quarter of 1 minus sin upsi.

Plot these up and what do you find. Sorry can we change these back to – can we change that to sin up si because my diagram will look better if I do costs up si, sin up si, costs up si. Right, okay, so obviously there's nothing in that there's just a change in the Figure 2 unfortunately, right.

Then what do we get we find that the right hand side looks like when up si is small the right hand side is looking like up si squared on 8 or something. Anyway it's rising quadractically, then it goes to 1 – this is pie by 2. Meanwhile the left side is basically a sin curve so we know what that looks like, it looks like this. So this is the left hand side, this is the right hand side and Bell has shown that the left hand side is smaller than the right hand side. So for only two values, smaller than or equal to the quantum mechanical results, the consistents with Bell's inequality for only up si is nought and up si is pie by 2. The quantum mechanical results violate this inequality for all values of up si basically.

So we conclude QM is inconsistent with these hidden variables. Once you've got a nice clean statement of this sort that there's – that quantum mechanics is inconsistent with something which EPR reasoned was – should be the case. Was very, you know, the indications were that it was the case you – clearly the right thing to do is to go out and make a measurement and allow nature to decide a view whether quantum mechanics is right or hidden variables are right.

So in 1972 this was first done using not an electron and positron pair but using pairs of photons that's usually how this is done. The analysis are slightly more complicated if you use photons than if you use spin half particles so we followed Bell in using spin half particles. But basically many of these experiments have now been conducted and the experiments vindicate the quantum mechanical predictions with the level of precision that you know that it's clear that the experimental results are inconsistent with hidden variables.

So the experimental results and that's from 1972 onwards there be many, always refined experiments are consistent with QM and inconsistent with hidden variables. So that means that quantum mechanics is not going to be replaced by a hidden variable theory at sometime in the future because you cannot construct a, you know, hidden variable theory along these lines is not going to be consistent with experiments that are already conducted so there's no point speculating about it.

So come back now to Einstein Podolsky and Rosen what are wrong with the arguments which indicate that somehow B's measurements knew about A's measurement. I think a lot of the – well sorry so the things that you should take away from this are first that when you measure something you do two things. You disturb the system and you gain information about the system. So when Alice measured that electron and found it plus a half for the spin in her direction A she disturbed the electron but she didn't disturb the positron because the positron was somewhere else and there wasn't – the positron couldn't possibly be disturbed by anything done to the electron until there had been time for a light signal to go from her operations to wherever the positron was. So she definitely doesn't disturb the positron but she does disturb the electron therefore she disturbs, she changes the state, she physically changes the state of the electron/positron system and that's why she's changed, she's collapsed the wave function from that linear combination to this here.

But she has gained information about B because of the correlation that existed in the original set up between her electron and the positron by know, by having discovered what was the state of affairs with the electron she was able to make some quite strong predictions about what B might find – what Bob might find on measuring the positron.

This experiment emphasises a theme that's quite common – it's quite a recurrent one in quantum mechanical calculations that it's very important to think holistically. To do this problem you have to think about the electron/positron system it's no good thinking "Oh I can deal with the electron." "Oh I can deal with the positron." Both together have to be considered because of these correlations in the system.

A lot of the confusion that I think Einstein, Podolsky and Rosen had and that is in many treatments of this experiment arises from slipping into the error of thinking that because Alice has found plus a half for the component of spin on her vector A that the spin is pointing along A. As we shall see a spin half particle has always plus a half of spin in the directions of all three coordinate axis. When you've made a measurement of the Z component you can know that the answer – you can know that it has a positive value for Sigma Z but you don't know – but you don't know what the values of Sigma X and Sigma Y are but you know that they have, you don't know the values but you know that they do have values which are comparable to that of Sigma Z. So what you should physically think of is that Alice is determined that the spin of her electron points in the northern hemisphere, well in the hemisphere that has her vector A for its pole.

She does not know it's pointing – that it's aligned with A. She only knows it's in the northern hemisphere of that. So if – when Bob makes his measurement and then she can say "Aha so I now know." She then knows for certain that the positron has its spin in the southern hemisphere of her vector A, right. Because… But she does not know where it points there because she does not know where her electron points in her hemisphere. She doesn't know where the positron points in its hemisphere. She only knows now – all she's learnt is which hemisphere the positron is pointing in.

So she can exclude as quantum mechanics says she can exclude only one result of B's measurement, namely if B chooses to – if Bob chooses to measure along the vector A then he will not find plus a half because the top hemisphere has no point in common with the bottom hemisphere and Alice knows that the positron is in the bottom hemisphere.

So the – I think the bottom line is that there isn't a logical problem if we just keep it focussed on the idea that what is pre ordained is which hemisphere the electron or the positron is pointing in not the direction. It's an error to think of these spins as pointing in in a particular direction. It's difficult to escape from the idea that a vector points in some direction but then it's difficult when we do relativity to get used to the idea that time is relative and that two events that are simultaneous – or one event that happens before another event for in our frame of reference in somebody else's frame of reference reverses the order of the events.

So the absoluteness of time is something that's very difficult to escape from but we all grow up, we get used to it. Time isn't absolute and quantum mechanics is telling us that no vectors don't point in particular directions they – there's a – in the case of spin half particles the best you can say is that they have particular hemispheres in which to point and will as we go on – so the next item on the agenda is angular momentum and that will enable us to look at this a little bit more closely about under what circumstances it is the case that a gyroscope or whatever seems to point pretty much in a definite direction. And we'll find in just the same way that things move only because they have ill defined energy things point in a definite direction only because they have ill defined angular momentum. Electrons do not have ill defined angular momentum they have well defined angular momentum and that stops them pointing in any particular direction.

Okay, all done.