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**Contributor** So where we got yesterday was that we had reduced the horrible partial differential equation in six variables which is posed by the time-independent Schrödinger equation for a hydrogen-like ion, to this equation here. Sorry – being premature...To this. So the internal motion had been separated from the translational motion of the atom as a whole. And we had exploited by using a change of variables, the centre of mass coordinates and the separation variable. And we had used the work that we did with the angular momentum operator earlier on to show that we could express the internal Hamiltonian in this format here. So this was called 'R' I think – that's right, for the internal motion.

Right., and then I pointed out that this Hamiltonian commuted with the angular momentum operators, because the only mention of the angles θ and  sits inside this total angular momentum operator. And therefore we can seek stationary states. There's going to be a complete set of stationary states which are simultaneous Eigen functions of 'E' and 'L', right? So we will also have the statement that L2onE and L = (L+1) E and L. And then when we're using these stationary states, the action of this operator will be replaced by this Eigenvalue.

And now we do end up with what I wrote originally, which is that we're going to have a Hamiltonian which has a subscript 'L' meaning it's the one that's valid for stationary states which have total angular momentum quantum number 'L', which is going to be PL2 / (2$\mu$ + L) (L+1) Hbar2 / 2$\mu$ (R2 - ZE2) /4ε0R. And this is fundamentally a Hamiltonian analogous, this is a Hamiltonian analogous to that, for the harmonic oscillator in the sense that we have now only one surviving coordinate. Of our six coordinates we're down to one, that's this here, the modulus of the separation vector and its conjugate momentum PR. So we've made an enormous simplification, made tremendous progress.

Now we're going to knock this into submission using the same approach as we did with the harmonic oscillator. We're going to define what will turn out to be a ladder operator AL, which is going to be defined to be dimensionless. It's going to A0 / $\sqrt{2}$. I'd better write this down from notes rather than my memory because these factors do matter. (IPR / $\hbar$) – (L+1) / (R + Z) / (L+1)A0, where A0 is the following not very helpful item 4ε0 $\hbar$2 / $\mu$ E2.

So this is an object I'm making convincing. Well let's make it convincing now. What is this object? This is the so called Bohr radius. So it was introduced by Niels Bohr before there was proper quantum mechanics using what turns out to be a fallacious model of hydrogen – the Bohr atom. And the way to see what its dimensions are, are actually to write – to bring this up on to this side and say look this is going to be E2 over 4ε0A0. And on this side A0's on the top, not the bottom. So therefore I have to say this is equal to $\hbar$ / A02 / $\mu$.

So out of this equation, by putting that E on the top and this 4ε0 on the bottom, dividing through by A02 to get this on the bottom so that this is something we understand to be, this is obviously electrostatic energy. And what's this that we see on the right side? Well $\hbar$K is momentum, right? So this is more or less P2 / mass. This is the reduced mass – more or less the mass of an electron.

So what we have on the right-hand side is P2 for K = 1 / A0. So if you have a wave which has a wavelength which is comparable to 1 / this scale radius here… Sorry, a wavelength comparable to this scale radius here – not to worrying about 2's at the moment – then what you have on the right to this side here is two times kinetic energy associated with the particle which has a wavelength which is on the order of A0. And what we have on the left side here is the electric energy.

So the scale that's being set, this dimensional quantity – the Bohr radius is the natural scale at which the kinetic energy associated with the uncertainty principle, the zero point energy – what with the harmonic oscillator we would have called the zero point motion, is on the order of the electrostatic energy. That's dimensionally where that number comes from. So, the thing to notice now is that AL is dimensionless. Why is it dimensionless? Well, this cancels the mentions of this. Obviously this cancels this. This is all dimensionless – this factor apart from A0.

And here we're looking at, again this is PR divided by, with this put on the bottom, $\hbar$ / A0, therefore $\hbar$K, therefore something with the dimensions of P. So this thing here, this operator here is dimensionless. Same as the ladder operators in the harmonic oscillator were dimensionless. Okay, so what do we do with that? What we do is calculate what A – and this thing of course carries the subscript 'l' because 'l', the orbital quantum number, is appearing in its definition. And what we do is we now calculate the AL dagger AL.

Right. So what does this give us? We have obviously an A02 / 2 up front because we're going to have two A0's and $\sqrt{}$2's. And then we're going to have, PR we showed was a Hermitian – well we engineered that was a Hermitian operator. So when I take the Hermitian adjoint of what I have up there I get a -IPR – the minus sign is from the I / $\hbar$. And then the rest of course is the same because it's Hermitian, well it's just boring numbers. +Z – well actually this is an operator, strictly speaking. We're in the position representation, so it looks like a number.

And that has to be multiplied on to IPR / $\hbar$. So no minus sign because this AL I'm writing down now. (-L + 1) / (R + Z) / (L + 1) = 0. So we have to multiply this stuff out. And the way we do it is we regard this in the back here as one factor, and that in the front as another factor. So this is looking like the product of a number minus some number, a number plus some number, right? So the usual pattern. So this is mirroring very closely what we did with the harmonic oscillator. So we get A02 / 2. These two obviously multiply together, and we get PR2 / $\hbar$2. And then these I have to multiply together.

So what do we get? We get the square basically of this number here. So we have (+ L+1) / (R2 + (Z / (L + 1) A02) - the cross-term here, which is going to be – well that'd be -2(2Z). Those are going to cancel, and we will have A0R. So that's the two easy parts, right? Because it's a front thing squared plus the back thing squared. And now we have to think of the cross-terms, which would vanish because this is (A + B) into (A – B) spiritually. These cross-terms would vanish if we weren't dealing with operators. And they fail to vanish only because we are dealing with operators, so there's a failure of commutation.

In other words the PR, well okay, so PR commutes with this. So for the cross-terms we don't get anything from this thing on this. But we do get something from this thing on this – namely we get the relevant commutator. So the extra term that arises because we're working with operators is going to be I / $\hbar$. That's that. So this minus sign and this minus sign cancel. We have the (L + 1). And then I have a PR, (1 / R). That's what I'm going to get from this term on this term not cancelling on this term on this term.

Okay. So what is this going to come to? Well let's rearrange things. A02 / 2 (PR)2 / $\hbar$ 2 + (L + 1)2 / R2 -, I'm going to put this one down next. 2Z / A0R. And then this term + Z2 / (L + 1)2A02. And while I have to do this, what I remember is something we handled already last term, that when I had to do – it arose when I was doing, when we were doing the commutator of P and V, the potential function of X. That turned out to be the rate dV /dX times the commutator of P and X. This is exactly the situation we have here, because this is the operator canonically conjugate to R. So what we have here is the derivative of (1 / R), which is… So -I(L + 1) / H x (- 1 / R2) – because that's the derivative of 1 / R – times PR, R.

But this is a piece of a canonical commutation relation. This is equal to minus I $\hbar$, right? We show that R, PR = IH. So in this order it's -I $\hbar$. So we have rather a load of minus signs, let me see. I think we have one, two, three minus signs, and another minus sign coming from here. So I think in total we have a plus sign. So this stuff here I believe comes to (L + 1) / $\hbar$ R2. Sorry, the $\hbar$ cancels – this $\hbar$ is on top, that's on the bottom. So we get an (L + 1) / R2. This can now be combined with this, except I've got the wrong blinking sign. Let me just double-check that. Yes I'm looking for minus. Is it minus?

Well anyway, I want it to be minus, so let's declare it to be minus. And I'm sure it is minus. Let's not spend time chasing down some wretched sign. Because now what we're going to do is combine this, so this is the side calculation here. We're going to have (L + 1) / R2 (L + 1) from up here. That's the (L + 1)2 minus one times this stuff. So you can see we're going to put this, and this is going to give me an L(L +1), which is exactly what I want. So this is going to be A02 / 2 PR2 / ($\hbar$ 2 + L)(L + 1) / R2 – (2Z / A0 R). And then I'm going to take this and join it on that.

So we get plus garbage. And the garbage term is Z2 / 2(L + 1)2. This should all be dimensionless. I think it probably is dimensionless on the grounds that it's the product of two dimensionless operators. Now what's the point of this ridiculous exercise? The point is that we should see the original Hamiltonians peeking out here. We should basically have our original Hamiltonian, plus garbage. So in order to get our original Hamiltonian we need to have a $\mu$ under here, and a $\mu$ under here. So why don't we multiply by a $\mu$ on the top and the bottom.

So this is A02$\mu$. And we want to take this $\hbar$ outside. And then that won't be under there as we want. We can allow that two to wander inside. And then this bracket becomes HL, this stuff here becomes HL. And we've still got unwanted garbage in the back. But that's exactly how it worked with the harmonic oscillator, remember? A dagger A was equal to H – the Hamiltonian – over $\hbar$ ω – $\frac{1}{2}$. There was garbage at the back, which in that case was a half. So this is obviously some constant with the dimensions of energy, and I'd better make it sure that I've done that right. It's the $\hbar$ – it should be squared, yes of course it should. Because it's the $\hbar$ I took out from there. Otherwise it is correct. And that's just the business.

So we've expressed H. Let's write this down in the other way. We've said that H = $\hbar$ 2 / A02$\mu$, which must provide the dimensions of energy – A dagger LAL - Z2 / 2(L + 1)2 – another way of writing. We've expressed H basically as dimensionless constant times as of A dagger, A dagger A. So this the harmonic oscillator trick. And it's all just looking a little bit messier. But this is only a boring number right? I mean what's the difference between a half and this? It's just a number. They're both just numbers, rational numbers.

Okay, so what do we do next? What did we do in the harmonic oscillator? We calculated A dagger, A – we found the commutator. That's was the next thing that we did, and that's what we do just right now. Make sure I've done the right… No, it's more convenient to work it out the other way around. Later use. Alright, so what is that? We have to write this horrible thing down again. So we're going to have an A02 / 2. Sorry, open a square bracket, because we're talking commutators now.

Write down IPR / $\hbar$, because I'm writing down A now, which is just there. I need to write down (-L + 1) / R. I don't need to write down the boring constant in the back, the z / (L + 1)A0. That will commute with everything in sight, and therefore will make no contribution to the commutator. Then I have to write down the corresponding parts of A dagger, which is minus IPR / ($\hbar$ – L) + 1 / R. Now I can rest easy. So this is what the commutator that I have to do. Obviously so PR commutes with itself – nothing doing there. PR on the other hand does not commute with this.

So we are going to get A02 / 2 I(L + 1) – that's that (L +1) – / $\hbar$. That's that H. PR, 1 / R commutator. I get that from that. Or at least live in hope that I do. What do I do with the minus sign? I put it in the bin. I shouldn't have done, right? There should have been a leading minus sign. Then I have the same term actually, because here I'm going to get plus. But everything is plus this, except I'm going to have a 1 / R, PR, which of course is minus this. So I'm going to get this all over again. So why don't you just rub out the two and then it's right.

What's this commutator? We've already discussed that problem. It's going to be the rate of change, going to be d/dR of this times the commutator. So this is equal to minus A02I(L + 1) / $\hbar$. And then I'm going to have a minus 1 / R2 for the derivative of this, times PR, R commutator, because that's how these things work. But this once again is minus I $\hbar$. So the two minuses here cancel, the two I's make another minus sign which kills this minus sign. All being well, this is equal to A02I(L + 1), the H is cancelled over R2. Check sign, yes should have a… No that's correct, that's correct. Good. Alright?

So we want to express this. Remember the commutator – so in the harmonic oscillator case what was this commutator? This commutator was actually one. So the bad news is, it's not looking very promising. At this point you think, "Oh no, it's not good because our commutator's some damned function of R." But we've seen that damn function a while somewhere before up in the Hamiltonian basically. I've lost the Hamiltonian, there it is. So we got L(L + 1) $\hbar$ 2 / 2$\mu$R2 is appearing in the Hamiltonian.

So supposing I would take $\hbar$ (L + 1), and from it I would take HL. Then everything in the Hamiltonian's would cancel except that middle term which has the right form. And namely it contains a 1 / R2. And what would we have? We'd have $\hbar$ 2 / 2$\mu$, I think, R2(L + 1) (L + 2) - L(L + 1). Did I do that right? Live in hope. Okay? So obviously there's going to be a factor of (L + 1) common. And then we're looking at the difference between (L + 2) and L, in other words 2. The 2 is going to cancel this, and this is going to equal (L + 1) $\hbar$ 2 / $\mu$R2.

So we can express, with this little side calculation we can go back up the board and express this as an appropriate multiple of the differences in the Hamiltonian. So it's going to be A02, that's this A02. Then we will want to multiply by $\mu$, divide by $\hbar$ 2. And then we'll be able to say these (HL - 1 – HL). Check that we haven't got any horrible factors in this. Okay. Alright. What was the next thing we did in the harmonic oscillator? Having got the commutator of the A and the A dagger, and having expressed H as a product of A and A dagger, the next thing we did was calculate the commutator – use these results to calculate the commutator of A with H. So that's what we do now.

I will do it here so that we can see our results. So I want to calculate the commutator AL, HL. And I can do that by expressing this HL, you see HL is basically a product of the A's. Now I've only got to locate the wretched product. It's at the top there, isn't it? It's so hard from this position to see what you need to see. This product etc. etc. etc. – this is the statement I'm looking for. I want that statement, alright? So this HL can be traded in for that product. So we have $\hbar$ 2 / A02$\mu$ times the commutator of AL and AL dagger AL.

And I can rest easy there. I've no need to put this stuff inside a commutator because it commutes everything inside. So it can't contribute to the commutator. So this is the commutator I have to evaluate. And this is easy-peasy, because this is the commutator of product `[[some 0:23:45]]` A with B and C, which in principle is the commutator of A with B, C standing idly by and then the commutator of A with C, B standing idly by. But A of course commutes with itself. So there's only one term, which is an AL commutator with AL dagger. So this is equal to $\hbar$ 2 / A02$\mu$AL, AL dagger.

And we just worked that one out. And the answer was here. Sorry, I did something wrong.

**Male 1** `[[?? 0:24:24]]`

**Contributor** Yes it's very important. I need an extra factor AL – thank you very much – in the back. This is standing idly by while AL works on his companion. Alright. So, I now need to plug this in for this commutator. And you can see that all these factors are going to cancel. This factor is going to cancel on this factor. And so we're simply going to get (H(L + 1) – HL) x AL I've been very helpfully told to include. So let me just... Right, so we're now in wonderful shape.

So we've completed all three steps of the simple harmonic oscillator calculation. And now we just need to go for the point of the exercise, which is so as we're given, we always were given that HL on E and L is equal to E of E and L, right? What we want to do is make ourselves a new – so we got one stationary state, we want to make ourselves a new stationary state by multiplying by AL, obviously both sides of the equation. So let me write down the right side of the equation first. This implies that E, which is a boring number, times AL, E and L = ALHL of E and L.

Usual business Swop these over. HLAL, which I'm not entitled to, plus the commutator so that restores order on E and L. We just worked that out and found that it was the difference of two H's times AL. So this becomes HL + 1 - HL, with an AL in back. Guess what? We have an HLAL here with a minus sign, and HLAL here with a plus sign. So the whole thing is equal to H(L + 1)AL, E and L. So we have achieved what we wanted to achieve. That is to say, we have shown that this state is an Eigen state not of HL but of H(L +1), and for the same energy.

So the map's looking a little different now from harmonic oscillator. But nonetheless we have a very powerful result. We have generated ourselves from a state which had energy E and angular momentum quantum number L. We've made ourselves a state which can only be characterised as E(L +1). We've made ourselves a state of the same energy, but more angular momentum. So what have we done? This operator, this is some normalising constant, right? We had just this kind of thing in the case of the harmonic oscillator.

So physically what have we done? Well, classically what have we done? We've taken an orbit that might look like this, and we've turned it into an orbit – I'm trying to make an orbit that has about the same semi-major axis and is rounder, if you know about Kepler orbits. So with the same supply of energy we've increased the angular momentum, so we've made the orbit less eccentric. So in the simple harmonic oscillator case what did we do? We made ourselves an orbit with less energy. And then we argued that we were able to show that the energy could never be negative.

So given this state now, we could apply A(L + 1), A sub (L +1) to this and make ourselves E, (L + 2), so that we get even more angular momentum. So like in the harmonic oscillator case we said we can make ourselves an orbit with even less energy. Is this possible with a given supply of energy, with a bound orbit, to have more and more, and more angular momentum? No it's not. At some point you've got the maximum angular momentum you can have for that given energy, which in classical physics is what would call a circular orbit. You completely destroyed the radial motion.

You see, what we've been doing here is we've been shifting kinetic energy, we've shifted KE from PR2 / 2$\mu$ to L2 $\hbar$ 2 / 2$\mu$R2, right? This was the tangential kinetic energy, this was the radial kinetic energy. We've shifted energy from here to here. When we've got no energy left in there, or as little as the theory, you know quantum mechanics allows, which won't be zero, but will be some amount, then we won't be able to shift any more. So there must be a maximum angular momentum for a given energy.

We'll call this $\ell$, alright? This is the maximum angular momentum and it is a function of energy. But we won't write it as a function of energy. We're going to find out what function of energy it is. So what does that mean? That means if we take the circularisation operator A$\ell$ belonging to this maximum angular momentum, and we use it on the state which has the maximum angular momentum for the given energy, what do we get? Nothing. That's the only way we can be prevented from getting a state which has even more angular momentum for the same energy, is if this operator simply kills this state.

So we've used this argument twice before, once with the harmonic oscillator case, and also in the case of the angular momentum operators. What do we mean by nothing? What we mean is the mod square of this is nothing. What does that map to? That maps to E maximum angular momentum A$\ell$ dagger A$\ell$E curly thingy is nought. Where have we seen a dagger A before? I think we must have seen it in the Hamiltonian. We need to replace that by the Hamiltonian times some horrible factors. Yes. All right. Well we already have it here.

So A dagger A comes right down to this line here. So this line here can replace the A dagger A in here. So we get to have the E$\ell$ on to A02$\mu$ / $\hbar$ 2. H$\ell$ + Z2 /2($\ell$ +1)2E$\ell$. Isn't much. But this thing, this is an Eigen function of this operator with Eigenvalue E. This is a boring number, so it stands by whilst this bangs into that and makes a one. This gives me E times this. And this is left over, it bangs into this and makes a one. So this implies that A02$\mu$ / $\hbar$ 2E + Z2 /2($\ell$ +1)2 is nothing. Or perhaps I should write this as equals minus.

So what have we done? We've got a relationship between the energy and the maximum allowed angular momentum. More than that, we know that these angular momentum quantum numbers, because this is orbital angular momentum we're talking about – we proved that those have to be integers. So, N being defined to be an integer. What integer? We know that $\ell$ is allowed to be nothing, 1, 2, 3, 4. So N = the numbers it's allowed to be is 1, 2, 3, 4 blah-blah, Nothing not included in the list because of that +1. So we have shown that E, the energy has to be of the form - Z2 $\hbar$ 2 / A02$\mu$ (1/N2).

So we have found the possible energies of a hydrogen atom – well in fact for a hydrogen-like ion. Because Z, remember is this integer which controls the number of charged unit on the nucleus. We have found this with the possible values, right? It's given by this constant which we know what it is – we'll simplify it in a moment – we know what it is, times over N2, where N is 1, 2, 3, 4. So this gives the energy levels. We write this as -Z2 x R / N2, where ŕ is what you see it to be, which is $\hbar$ 2 / 2 A02$\mu$, which is not very intuitive. The way to make this intuitive is to take those A0's – there are two of them – and make one of them back into its $\hbar$'s and things.

Now where did we define A0 for heaven sake? It was right over here somewhere, right? – there it is. So one of those two I'm going to replace by that garbage there, alright? So this is going to become, on the bottom you're going to have an 8ε0A0, alright? That's the 4ε0. The $\hbar$ 2 will cancel top and bottom, so that goes away. The $\mu$ and the E2, well the $\mu$ will go away with this and the E2 will sit on the top. So the [Rietberg 0:36:06] is what? E2 / 4ε0A0 would be the potential energy of two electric charges that were separated by A0.

So this is half of the potential energy at a separation of A0. So this is the fundamental energy scale of atoms. And what does it equal to? 13.6 electron volts plus 13.6 to three significant figures. So the energy range at which we work, the battery that you stick into your camera or something, has 1.5 volts basically because of that 13.6 eV's. It's all of condensed matter physics is a mere reflection of that number. Rather you know, that's why we live at 1 eV, not at 1 meV, or 1 milli eV, or whatever.

So, what do we need to do next? Yes, jargon. This is called the principal quantum number. So in these hydrogen-like ions we've discovered that there are a whole series of distinct states which have the same energy at different angular momenta. So let's talk a bit about degeneracy. Okay, so if a principal quantum number N = 1, we have the $\ell$ = N - 1 = 0. In other words, the largest angular momentum you can have is nothing. And in the ground state of hydrogen there's one electron. It sits in the state with the lowest energy, which is going to be associated with N = 1.

And it has no angular momentum, it's on a totally radial orbit in classical physics, right? Not going round and round at all. It just goes in and out, in and out – I mean in classical physics, quantum mechanics. But it doesn't have an angular momentum. So that's a surprising result. For N = 2, the maximum angular momentum $\ell = 1$. That means that $\ell$ can be naught if you like, and $\ell$ can be 1, right? This is the maximum angular momentum.

So there's a slightly funny thing going on here. N was introduced as 1 + the maximum angular momentum. But now I'm saying when one standardly thinks about it, when one thinks what's the value of N, from it, one takes this N -1, the maximum angular momentum. So in this sense we have one state. It'll be two states. Here basically we have – this is for a spin-less particle, right? It'll turn out to be two states when we include the spin of the electron.

But remember we were doing the gross structure, which means we said we were going to forget about the spin of the electron. Here we would have one state, and here we would have three states, right? Because for L = 1 we got total angular momentum one. Which means we got three possible orientations of it. N can be one, nothing or minus one. So we have three quantum states here, one here. So we got four states all with the same energy for N = 2, one N = 1, and so it goes down the line. So the number of states is increasing rapidly.

Because there'll be five states for N = 3, the maximum angular momentum will be two. For two units of angular momentum you've got five possible orientations. And then you've still got three of these and one of those. So that's nine states etcetera. So the structure we've derived is extremely degenerate. What does this have to say about experiments? So stick some hydrogen atoms in a vessel and pass an electric current through, and get the electrons knocked out of their comfort zone, and you will get photons coming out at discrete frequencies.

$\mu$ is going to be the difference in the energies over H, which is going to be Z2, the Rietberg constant / $\hbar$, 1 / N'2 – 1 / N2. This is for N goes to N'. If you're in one of these higher states, for example N = 2, you will have less, your energy will be a smaller negative number, right? You'll have 1 /22, you'll have a quarter here. And this will be – if you could then fall down to the state N' = 1, in which case this would be one. So this bracket will be say three quarters, and you'd get three quarters of this number coming out. So that gives you some frequency.

And what we have is series of lines. Or the way we think about this is that we have a series of lines, each fixed N', i.e, bottom level. So if we fix N'1, we can have transitions from N is 2:1, or N is 3:1, or N is 4:1. And these are the successive lines of the `[[Liman 0:42:51]]` theories. So here is the energy of N = 1, here's N = 2, here's N = 3. And Liman Alpha is the name used for the spectral line associated with an electron tumbling from N = 2 down to N = 1. And Liman Beta is associated with from N = 3 down to N = 1, which is further to fall so it emits more energy. So the line appears at higher frequency, shorter wavelength.

So the Liman theories is for N' = 1. If N = 2, we're looking at Liman Alpha – that's what it's conventionally called. If N = 3, it's Liman Beta. And this has 112nm, is that that? 121, sorry. And as you go down to N= infinity, in other words if you fall all the way from not being bound into the bottom of the atom, then this is the beginning of the Liman Continuum. And that's 92nm roughly speaking. I've got a more accurate number here – 91.2nm. So these lines are all vacuum ultraviolet lines. They all carry, so this one is carrying 13.6 eV of energy, and this is carrying three quarters of 13.6 eV of energy.

So they're carrying enough energy to kick electrons out of the air molecules. So these photons are absorbed by all kinds of things. These are very easily absorbed photons because they carry enough energy to lift electrons out of most atoms. And then we have the next is the `[[Balmer 0:45:03]]` Series, which is where the whole story started. Which is so N' is 2, N = 3. If you go from 3 to 2, that's called Balmer Alpha, but it's written as H because that stands for hydrogen-alpha.

So Balmer was a Swiss schoolteacher who empirically fitted the formula we've derived. I've lost it – there it is. He fitted that formula empirically to measure frequencies of lines that he identified as being the Balmer Series Lines, well a series of lines which is called the hydrogen series. So this is called H, and it's a pink photon. It's 656nm. So it's pink light. So, many astronomical objects are pink because they are shining in H, in the Balmer Alpha. This is Hβ etcetera.

If you, then you go to Passion. That's for N' = 3. And obviously N can be four or five etcetera, etcetera, etcetera. So these start off as pink and they get bluer. So as you go down this list, the wavelengths get shorter. As you go to infinity – where's the series limit? I did write it down here, 364.6nm. So they go from pink light right through the rest of the optical spectrum, to the ultraviolet region. And the Passion Series starts at 1875 I think. So N = 4, you're looking at 18, yes. These are sort of more or less optical.

By now we're in the near infrared etc, etc, etc. So, that's pretty much the right place to stop I think. What we should do, just one other thing I would point out is that so you can apply these formulae to the innermost electrons of other atoms, atoms that have more than one electron. You can't apply them to the outer electrons with any useful way, because we've done all this right with no other electrons present. We've got one nucleus and one electron.

But there's one very important thing to take home, which is that this energy scale goes like Z2. So the characteristic energies of the innermost electrons are going up like Z2. By the time you get to uranium which has 92 units of charge, so Z is 92, you're almost a factor of 104, you're almost a factor of 10,000 higher in energy. Which means that these electrons are moving essentially relativisticly. So that's just a thing to bear in mind. Okay. And we'll look at the wave functions that go with this lot, tomorrow.

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