Title	*003 Operators and Measurement*
Description	Third lecture of the Quantum Mechanics course given in Michaelmas Term 2009
Presenter(s)	James Binney
Recording	http://media.podcasts.ox.ac.uk/physics/quantum_mechanics/audio/quantum-mechanics3-medium-audio.mp3
Keywords	physics, quantum mechanics, mathematics, F342, 1
Part of series	*Quantum Mechanics*

Contributor This is where we were. I had asked you to take on trust as a result that will be derived some way down the course, these results which express the state in which you are – the spin-$\frac{1}{2}$ particle like an electron – is certain to have a $\frac{1}{2}$ plus a $\frac{1}{2}$ for the answer to its spin along the unit vector n which is given by the polar angles theta and phi.

This particular state in which quote un-quote, its spin is along n. Now somebody asked me about this at the end of the lecture and I have to remind you of the health warning I gave in the first lecture which is that we talk about the spin being along a certain axis although even when you know the answer to the spin along z is going to be a $\frac{1}{2}$ and not minus a $\frac{1}{2}$ - and you can only get those two answer, a $\frac{1}{2}$ or minus a $\frac{1}{2}$. Even when you know it's plus a $\frac{1}{2}$, it doesn't mean the spin is really along the z axis. There's still a substantial component of spin in the x-y plane and you do not know its direction.

So we use this loose talk – the spin is along some particular direction like the z axis or this n – meaning it's a shorthand for you are certain to measure plus a $\frac{1}{2}$ if you measure the spin in this direction.

Right. So with that health warning, the state in which you're certain to measure spin a $\frac{1}{2}$ in the direction n is this linear combination of the state in which you're definitely going to get minus a $\frac{1}{2}$ along the z axis and this state in which you will definitely plus a $\frac{1}{2}$ along the z axis.

And similarly, the state in which you're definitely going to get minus a $\frac{1}{2}$ along the n direction. Unit vector n is this other linear combination of those same two states of well-defined spin along the z axis.

So I asked you to take that stuff on trust. And then we did some stuff with that. And I hope I persuaded you that these formulae are not completely implausible in the sense that what we did was we calculated the probability. If it's definite – if we know the spin is plus a $\frac{1}{2}$ along the z axis, we calculated the probability that we found plus a $\frac{1}{2}$ along the n axis. We found that that probability was in fact simply cos squared theta upon 2. And this behaved in a reasonably plausible way in the sense that when it was 1 when the n direction was the z axis, and it went to nothing when the n direction was the minus z axis and other such good stuff.

Then I wanted to show you this. All I wanted to show you was what these formulae predict for what the state is of definitely having plus a $\frac{1}{2}$ for the answer to what's your spin in the z – in the x direction, right?

It's easily done. Because we have the formula here. I was thinking I was trying – for some reason it went into my mind that I had to derive these formulae. And we didn't have the bits on the table to do it.

So all we have to do is plug into those formulae that theta's pi upon 2 phi is nought. That is by definition of polar co-ordinates, makes n the x – the unit vector in the x direction – which I'm calling ex.

And put in pi – if you put in pi upon 2, then you're looking at sine pi upon 4, cos pi upon 4, 1 over root 2. And phi being nothing, means those two exponentials are nothing.

So the state of having your – the state of having your spin definitely down the x axis, in that sense, right? Again, with that health warning. So the strict statement being that we are guaranteed to get plus a $\frac{1}{2}$ if we measure the spin down x. It turns out to be just the sum – essentially the sum – of these two states.

That's not very exciting I think.

Let's put in – but now let's put in theta is pi upon 2. Phi is pi upon 2. Which, by the definition of polar co-ordinates makes the unit vector n the y direction. Then what happens?

Well, what happens is that those e to the i thi is upon 2 become e to the i pi is on 4. And if I take the first e to the i pi upon 4 out then the second one – so this cosine is 1 over root 2 again which we've taken out. But this one becomes e to the minus i pi upon 4 twice. I.e. e to the minus i pi upon 2. Which is actually minus i. I'm slightly worried by this. But never mined. The sine is of no importance. i thought it was a plus i but it is looking like minus I at the moment. So maybe just minus i. It's of no importance.

What matters is that this state, which is physically quite distinct from this state, is also a linear combination of these two. And the probability of if you are in this state, the probability of measuring your spin along z to be minus is going to be a $\frac{1}{2}$ because of this 1 over root 2.

So the complex number which comes here has the same modulus as the complex number comes here. And ditto here. But this complex number is the same as the complex number which appears there in modulus. But different in phase. So that what – the crucial point is that the -

Right, we are working within a formalism where we're saying the state of my system can be written as a minus minus plus a plus plus. And we understand that these things are the probability amplitudes to measure spin down on z, given that my system is in this state. And this is the amplitude to measure spin up on z, given that this is my state of my system. Right? That's the formalism we're working with.

And you might think that it's only the modulus of these complex numbers that matters physically. Because the probabilities are obtained by doing mod square of them. But this example is showing you that that's not the case. The phases of these things are vitally important as well.

But that's a very quantum mechanical thing – that the complex phase – the phase of the complex number – encodes crucial physical information. Is the spin of this particle more or less down the x axis or more or less down y axis is controlled by the rati- – by the phase of this animal, relative to the phase of that animal.

Let's do another example of a physical system which is a two-state system. Let's talk about polarised light. This is an example which enables us connect back to classical physics in an interesting way.

So let's do classical physics. Well you know all about polarised light. Well, actually, you may not quite because it may be part of upcoming emag course. But you will recognise enough of it I think.

I can write the electric field – supposing we have a – we have y direction way, the x direction this way. Suppose we have polarised light with the electric vector in this direction with that angle being theta, then we can say the electric vector is equal to some number in front of cos theta times ex plus sine theta times ey times cos. And we get T.

Now supposing we – so we've got an elec- We understand we're writing down the electric vector of an electromagnetic wave. A plain, polarised electromagnetic wave that's travelling in the z direction. Okay? In some plane. This is what it looks like. It oscillates with some frequency – angular frequency – omega.

Right. Now supposing we stick in some – this beam comes along and it hits some Polaroid. And let's imagine that the Polaroid blocks the electric vector. So Polaroid blocks one of the polarisations. Let's orient our piece of Polaroid so it kills the oscillations parallel to the y axis. And lets only through the oscillations parallel to the x axis.

So after Polaroid we're going to have that e is equal to e nought e nought cos theta cos omega T ex.

It just wipes that out.

So the intensity of the intensity of the radiation – the energy that it carries – is going to be looking like e nought squared times cos squared theta. Yes. And `[[strictly speaking 0:09:47]]` we should really do a cos squared omega T average value, which is in fact a $\frac{1}{2}$. But I don't think we're really interested in that.

The crucial thing is that the intensity of the light that gets through is going to be moderated by the square of the cosine of that angle. Right? The angle between the electric vector of the wave and the direction that the Polaroid lets through.

That's what classical physics teaches us.

How would we express this? So let's now think about this from a quantum mechanical perspective.

What classical physics says is an electromagnetic wave, quantum mechanics says is a stream of photons. And each photon encounters that Polaroid on its own. On it's lonely ownsome. And either it's killed by that Polaroid – turned into something else. Destroyed. Or it's allowed through. It can't be half allowed through or cos squared theta allowed through. It's either allowed through or it's not allowed through.

So how does that look? What we say is the state of incoming photon, we can write as a linear combination. We can say that this is equal to cos theta of a state in which it is going to get through, Because, in some sense, its electric field is down the x axis, plus sine theta of a state that is not going to get through.

So this is the state of certainly gets through. And this is the state of certainly blocked. We're taking the position that the Polaroid is making a measurement on the photon.

So what's the probability gets through? Well, it's equal to – this is an amplitude – right? It's equal to the amplitude for getting through mod squared which is equal to cos squared theta. And therefore, the number of photons that gets through is proportional to cos squared theta. But the number of photons is the amount of energy that gets through. Right? So it should be the intensity of the light goes like cos squared theta. And quantum mechanics recovers our classical result.

We can go further than that because we know that – if we think about circular polarisation.

So we know that classically, we can write the electric field of a circularly polarised radiation. So in plain polarised radiation, the electric field just oscillates up and down some definite direction. In circular polarisation, in a given place, the electric field always has the same value. And it rotates in its direction. So now it's pointing in the x axis, now it's in the y axis, now it's pointing in the minus x axis, etc. And it can go around clockwise, or anticlockwise, etc.

How do we write that classically? Well, we can write that it's e nought over root 2. And then I would write the real part of – the neatest way to write it is the real part of ex plus iey times e to the i omega T. And that's all inside this real operator.

Let's think about that for a moment.

Because what does that give me? This ex meets that cos plus I sine. So we find, when this real operator works, and we're looking at ex times cos omega T. And this iey meets cos omega T plus i sine omega T. So this i and the i that's sitting inside here, make the real part of this minus sine omega T. So this is looking like ex cos omega minus ey – these are unit vectors – sine omega T. And so that's what we get from this notation.

So this indeed is a circularly polarised beam. The mod square of this electric field is going to be e nought squared over 2. And it's, in fact, right-hand polarised. In that complex – in the plane, it's going to go around that a-way. Because y is going to start – is going to become negative first. The component because of this minus sign. Right?

And similarly, if we wrote – so let's call that e plus for – you know, e subscript plus – for the electric field associated with the right-hand circular polarised beam. Correspondingly, we would have e minus for a left-hand circularly polarised Johnnie – would be this.

We get the – we get a change in the sense of rotation just by changing that plus i to a minus i. It's easy to check that that's true. So this is left-hand polarised.

How would we do this quantum mechanically? What we would do is we would say that there's a state plus which is equal to the state that has its electric vector in the x direction. Plus I times the state which has its electric vector in the y direction and doesn't get through the Polaroid. And this does get through the Polaroid. And we would say, so this would be a state of – this would be right-hand polarised state of our photon is a linear combination – and when I should have a 1 over root 2 outside here. That's that what root 2 basically.

So a state of circular polarisation of a photon is a linear combination of two plain polarised states. And similarly, we would have that minus is equal to the left-hand polarised state would be.

And we would be able to make statements like, if we want a kind of statement we could make is we could add these two equations and we would discover that being polarised in the x direction is 1 over root 2 of being right-hand polarised plus being left-hand polarised.

And this is also a result that we have in classical physics. That if you have a plain polarised beam, you can consider it to be a linear superposition, if you like, an interference pattern, whatever, between two circularly polarised beams of opposite senses of polarisation.

But there's a different – but this has a different meaning. Sort of emotionally. Right? This is saying that a particular state of one photon, of a particular photon, is this linear superposition of its two other possible states.

Something else that you learn from this – I mean another thing that should be pointed out – is that in classical physics we were using – I was using i here and up there as a sort of handy way to reduce algebra etc. If there was a real operator sitting in front of it, the electric field was totally real, and any appearance of the square root of minus 1 was merely as a short hand, as a trick, as a device for compressing the algebra.

In the quantum mechanical case, this i is i. There's no real operator. There's no nonsense with that. This is inherently a complex animal.

Now, maybe it's time to move across.

Let's say a little bit about measurement. We've already encountered these ideas really but let's – let me take you back to what we did yesterday with the energy representation. What I said was, look, supposing I write upsi in terms of some basis vectors, i. Because we had agreed that the quantum state of a system, a ket, was an inhabitant of a vector space. Vector spaces have basis. Therefore, any ket can be written as a linear combination of basis vectors.

Supposing these happen to be, physically, the amplitudes to measure a particular value of the energy, say ei, then I hoped I'd persuaded you that the physical meaning of this ith basis vector is the state in which you are certain to measure ei. Because, because if upsi is the state ei in which we are certain to measure this energy, then what does that mean? It means that ai is 1 and aj – and every other a – has to be nothing. aj not equal to i.

And so we can look into this expression here under those circumstances. Under those circumstances, upsi on the left here becomes ei. This sum collapses just to i. And that tells us that i is actually the state in which you are certain to measure ei.

So that's how we understood the meaning of these things here.

Now, suppose that upsi is some general thing. It's some general state. In other words loads of these ais are non-zero. So it's some superposition linear combination of a non-negligible number of these states of well-defined energy.

So suppose, initially, that upsi is not a state of well-defined energy. It is a sum ai ei with lots of non-zero ai.

Fine, now suppose we measure the energy. If we measure the energy according to our conception – well obviously. If we measure the energy, we are going to find one of the allowed values. One of the values in the spectrum of the energy, we're going to come up with one of these eis. Shall we call it ek?

So, we do a measurement. So we measure e and find ek. Having found ek, we know what the energy is. We know it's ek. Therefore we know the state of our system. So now upsi equals ek.

So after we've made the measurement, upsi is different from the ket that it was before we made the measurement. It's changed into this. Which is just one of the terms that occurred in this series. So this sum ran over many of these ais. And one of the i's was k. It just happened when we made the measurement, bingo, this is the one that popped out. But having made the measurement, we know what the energy is. It's ek. So the system is definitely in the state ek.

So the original wave function is of a state – a quantum state – is changed into a different quantum state on making the measurement. And this different quantum state looks simpler than that one. And what we – what people conventionally say is that this quantum state, as a result of our measurement, has collapsed into this quantum state. So this is the collapse of the quantum state.

Traditionally known as the collapse of the wave function. But we haven't yet met wave functions. But it's the same phenomenon.

Now, it's in the extremely interesting question, what's really happening here? This is a fundamental, absolutely non-negotiable piece of this theory. The matter's discussed rather more in chapter six of the book. And, at some point, say in the vacation, I would urge you to read that. And you will find that it is – this piece of the theory is fundamentally unsatisfactory. It's clearly not right. But nobody knows how to – there are various proposals, including many worlds and all sorts of things for fixing it, but none of them really – there is no known, satisfactory fix. There is no consensus. There is no really persuasive fix.

Consequently, different people say, "Well I think the fix is probably something like this." Somebody else says, "The fix is something else like this."

The fundamental principle that I think everyone will agree on is A) it's non-negotiable. It's absolutely essential for the working of the theory that we do some such collapse.

Two) that when you make a measurement, there are – logically there are two possibilities. Logically, it's just a thought process. Right? Okay, I was, I wrote that down because frankly I didn't know what the energy was. So that covered my basis. And, you know, it was probabilistic. There were many possible values of the energy, etc. that were – and I stuck in some amplitudes to reflect my uncertainty.

And, having made a measurement, I discovered what the energy was and so it's this and now everything's okay. We've discovered something. So I've updated my information. And the state vector is merely reflecting my improved information. It's a subjective change. Not a real change. That interpretation proves to be untenable.

There really is a change. It's a – at the moment we're operating – chapter six – only chapter six, introduces an apparatus that deals for muddle and uncertainty, which is kind of worrying. Because in real life, and real physics, there's always masses of genuine uncertainty and genuine muddle.

But we are not – we are operating in an ideal world at the moment in which there is total clarity, there is complete information, there is nothing left to chance. Beyond what is inherent. I mean -

So this is the well-defined state of the system. It changes into some other completely well-defined state of the system. It actually objectively changes. And here we have a crucial thing that is being added in quantum mechanics to classical physics which is the concept that, when you make a measure, you disturb the system that you're measuring.

I think this is totally reasonable. It's obviously an abstraction that classical physics makes that you can make measuring instruments of arbitrary delicacy so that you can have these – so when a measuring instrument interacts with a system, the measuring instrument in classical physics is affected. You know, the needle moves over or a light glows or whatever. But the system carries on blithely, you know, without being changed in any way.

It's clear. There's action and reaction. If the instrument is affected, the system is affected. And since we're concerned with systems - quantum mechanics is about systems which are very small – it's very natural that the impact on the system should be kind of substantial.

So it's totally reasonable that we should be working with the theory where every measurement is associated with a disturbance of the system and leaves the system in a configuration different from the one that it found it in.

So that's not the problem. The problem is that the theory doesn't describe the process of getting from here to here. But that's a topic which I can't discuss at this stage. Or, indeed, in this course. It's – you can find something about it in chapter six. It's all highly off-syllabus.

I want however to point out something else which is that we started with a basis up there. Remember we started with i and the mathematicians already taught us to associate with i – the ket i – a bra i, such that ji equals delta ij. So in our physical example, this maps into ej ei equals delta ij. So this was just mathematics. But it has a deep physical meaning as follows.

I think I made the point yesterday that if you want to know what ak is, the way to find it is to do ek upsi. That's, broadly speaking, why we introduce these bras. We introduce these bras because we wanted, out of an object like upsi, to extract the amplitude for something to happen. Because, you know, amplitudes are the things whose mod square make predictions and we, you know, that's what we take down to the lab to test against nature.

So let's ask ourselves, in this context, let's look at this formula. This is the amplitude to find energy ek, if the system is in the state upsi. So what's this? This is the amplitude to find ej if the system is in the state ei. Well, if the energy is ei, it can't be ej, can it? If j's different from i.

So the reason this thing vanishes, when i is not equal to j, is because it reflects the exclusive – well, it reflects the fact that if your energy is ei, it's ei. It's not ej.

So this orthogonality condition is a logic necessity. The mathematicians have given it to us. But we need it for physical reasons. We need it – it's associated with our – it's a requirement of our fundamental principle that this gives us the amplitude of measuring ek.

Some other little details I should cover at this point is, suppose we've got upsi upsi is equal to sum ai sum of some basis i. It might be the energy of the states. Might not. Who cares.

And suppose we have some other state – we have some other quantum state – which is the sum bj j. So these two states are two different states because they have different – they're associated with different amplitudes. This is associated with a set of amplitudes, ai. The numerical values ai. And this state is associated with the numerical amplitudes bi bj, whatever.

And let's calculate the number phi upsi. So we have to take the complex – we have to make the bra out of that and use our rules that we introduced yesterday. So this is the sum j of bj complex conjugate j times the sum ai i. But when i meets j, we get a delta ij. Which means the – and when we conduct the sum over i, we get nothing except there's only one term that contributes because of the delta ij. And that's when i equals j. So this becomes the sum bj star aj.

So that tells us how to work out this complex number in terms of these quantum amplitudes. That turns out to be very useful. The thing I want to say at the moment is, supposing I worked out the other thing. I worked out phi – sorry – upsi on to phi instead of phi on to upsi. Then everything would be the same here except that this will be an aj star and that would a bi. And we would be looking at the sum over j of aj star bj.

But this is the complex conjugate of this, by the rules of all complex, because this is just a sum of complex numbers, right? And we know what the rules of complex conjugation are. So this is the sum bj star aj star. Which is, phi upsi star.

So this is an important equation to remember. That upsi phi is equal to phi upsi complex conjugated. You'll need that many times.

I think that's all we want there.

Let's now introduce the next topic which is operators and their connection to observables. Things we can measure.

So what – we're interested in linear operators. What does that mean? I guess you probably know but let me just write it down anyway.

So lets – if q is a linear operator – Well first of all, q is an operator. What does that mean? That means it turns kets into kets. Give it a ket, it produces a ket. That is to say phi – if I do q, the operator, on upsi, the ket, I get another ket phi. Right? That's what an operator is. It's something which turns kets into kets.

What's a linear operator? If I have q on a linear combination of alpha upsi plus beta, say, of chi – so this is just two, any old two kets – I take alpha times one and beta times the other because I know I'm allowed to do that. Well what is that? That's equal to alpha q operating on upsi plus beta of q operating on chi. That's the linearity property.

So we're only going to be interested in these linear operators.

Now, let me write down an operator. There's an operator – a very, very, very important operator – like this. If we have a basis of kets i, I can form this creature here. This is the ket i somehow multiplying the bra i. Just like that.

The first thing I have to do is persuade you that this is an operator. Right?

So I say, let's consider this. And I need to persuade you that this is an operator. How do I do that? I show you how it operates. Right? So as long as I – if you know how this operates on any ket, then it's an operator.

So let's have a look at this. Supposing I do i upsi. What does that give me? It gives me the sum over i of i i upsi.

Now, this is a complex number. It's even an interesting complex number with emotional appeal. Because it's the quantum amplitude for something. But let's not worry ourselves about that at the moment.

This is a complex number. This is a ket. So this is a linear combination of kets. Ergo, it is a ket. It is something. We can call it phi if we want to. Alright?

So that means that i does turn upsi into some state phi. It is an operator.

Now, let us replace upsi with -

Let's replace this upsi with its expansion. Sum ai i. So I can write this as the sum i on – this is going to work on – the sum over j of aj j. So this is another way of writing upsi. We've done it time and time again.

Now this I is going to meet that j and produce a delta ij. When I do this sum over j, every term will vanish, except the term where j equals i. And then when j does equal i, we'll have i on i, which is 1. So this is equal to the sum i i which is upsi.

So i, this operator, this thing here, is not only an operator, it's the identity operator. Because it turns upsi – any upsi gets turned back into itself.

So we have that this thing here is the identity operator. And I've told you nothing about what these things are except they form a basis; they're a complete set of kets.

And we use this representation I, sometimes called a resolution of the identity. It's not a phrase, expression I will use. But we use this representation of the identity operator time and time and time again. It's tremendously valuable.

Now let's introduce a sexier operator. In fact the most important operator in the universe. We're going to introduce h which is, by definition, the sum ei, ei, ei.

So these are the states of well-defined energy. And the numbers ei are the possible energies. They're the spectrum of the energy operator. This operator here is called the Hamiltonian. After William Rowan Hamilton who lived in the first half of the 19th century and introduced the classical analogue of this.

And, I think, I hope it's clear from what I did above, that this is an operator. That is to say, if we do H upsi, we will get this stuff times e upsi. This is a complex number which multiplies this real number times sum kets. So we will get back a ket. And it's obviously an operator associated with the energy.

And the general scheme is going to be, with everything that we can measure, we're going to associate an operator. And we're going to do it in just this way.

But let's just focus on this particular one for a moment because it is the most important operator in quantum mechanics.

And let's see one thing that we can do with this. Supposing we work out upsi H upsi.

So, what is this? This is a number. A complex number. Why? Because H operating on upsi makes some state, shall we call it phi. And upsi – the bra – working on phi, produces a number. So we can see the state – we can say straight off that this is some complex number.

Let's find out which complex number by putting in for upsi its expansion, sum ai ei. So what we're going to do now is replace both of these by their expansions.

So on the left here I'm going to have aj star e ej. So that is upsi, the bra. Then I have H. And then I have ej – sorry – then I have the sum ai ei. And H itself – oh dear this is getting complicated – is the sum over k of ek k.

Sorry, ek, ek. So every term in this expression here has been expanded in terms of the basis states – the states of well-defined energy. States where measurement of energy is certain to yield a particular result.

Now what happens? We're summing over every thing. j is being summed over, i is being summed over and k is summed over.

This ek – oops, yes, that's right. Sorry. Let's work on -

This ek is going to – this is a – this ek is a linear function on this. So this passes through the ai by the linearity of this function and meets this and produces me a delta ki. So when I sum over i, I get nothing except when i is k. So that is going to – this sum is going to collapse.

In the next line, we're going to be looking at the sum over j of aj star ej. Oops. Which way it's pointing? It's point this way. Sum over k ek ek. And the action here is that this sum over i is going to make that into an ak. So that's looking this way. That is an ak. Because of the orthogonality that and that.

Now we repeat this trick. We say, when we do this e, j is going to pass through this, by linearity and etc. Meet that and produce a delta jk. So we'll get nothing on this `[[sum 0:43:34]]` except when j is equal to k. So this is going to become the sum over k of ek. This is – this a star after j is going to be – the only term that's going to survive when this meets this is when j equals k.

So this is going to become an ak star. This ak star is going to meet with that ak to produce ak mod squared.

But ak mod squared is the probability of getting this energy. So this is equal to the sum of Pk. The probability of getting ek times the value of ek. In other words, it equals the expectation value of the energy.

So that's one reason why these operators – why operators associated with observables in this way – are so useful. If you want to know – your state is upsi and you want to know what the expectation value of the energy is, you take H, the operator associated with energy, and you squeeze it between the bra upsi and the ket upsi.

So we can – you give me any observable. Call it q. For that observable, we have agreed there will be a spectrum. So there will be numbers qi. The spectrum. The possible values that the measurement can return. And there will be a complete set of states, qi. These are the states for which the result of observing q is certain.

And what can I do? I form the operator q. We might give it a hat, just to stress that this an operator which is a mathematical object. Whereas q is a sort of concept like momentum or angular momentum or position or something. We've got an actual operator which is defined to be the sum over k of qk qk qk.

And by exactly the logic up there, we will find that the expectation value of the observable q is going to be upsi q upsi.

So that's just repeating what I've done for the energy operator – the Hamiltonian – making the point that it's going to carry through for any observable.

Now there's more than – we can do m ore than that. We can now ask ourselves, okay. Let's look at this.

What is q operating on qi? What does q – q is an operator. What does it do to one of the special states associated with that observable?

Well, by definition, this is the sum over k of qk qk qk qi. Right? But this is merely the definition of the operator q. There is what we're operating on. This is going to produce a delta ki. So when we do the sum over k, we get nothing except when k is equal to i. So what we get – only there's only one term in this series survives. And the answer is, it's qi qi.

So what q does to this state of well-defined q – the well-defined value of the observable – is it turns – it makes a scale model of what we started with. With a scaling factor qi. So, in the mathematical language, which I guess you've met, this says that qi is an eigenket of q. You have met that, right? You've met eigenkets. Eigenvectors, whatever, of operators.

And the eigenvalue – so that the states of well-defined observable, which are really the primitive, physical thing, turn out to be eigen states of this operator that we've introduced. And the eigenvalues are the possible values of the elements of the spectrum; the possible numbers you can get if you measure the observable q.

Well that probably is the right moment to stop.