Title	*007 Back to Two-Slit Interference, Generalization to Three Dimensions and the Virial Theorem*
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Contributor Okay. So now we've investigated the states of well-defined momentum, which as you recall were these plain waves with a wavenumber, p upon, momentum upon h-Bar, we can go back to this problem of two-slit interference. And ask ourselves why it is that quantum interference isn't observed in macroscopic objects. And also assess what experimental setup would be required to see quantum interference with say electrons.

So we can put some numbers in to this experiment. So as you recall we had some gunhere that was symmetrically placed with respect to a couple of slits here in an obscuring stream, screen. It fired out particles. Some of the particles got through the holes. And they came to here and we said that the amplitude up here would be the sum of - the quantum amplitude to arrive at this point would be the sum of the amplitude to go via this route, or to go via this route.

These routes, well the distance from the gun to the two slits by set-up is symmetrical, is equal so any difference in the amplitude that they come there is to do with the change in the amplitude when it goes along this route as against along this route. So that d plus would be the distance from the upper slit to this place, and d minus the distance to the lower slit.

Then we can write a formula that d plus by Pythagoras's theorem, is going to be l squared plus x minus s squared square root, and correspondingly d minus is going to be the square root of l squared plus x plus s squared.

And we make the reasonable conjecture, I mean the right way to think about this is these particles; let's imagine that our gun here has been tuned to emit particles with some well-defined energy. That means that as they go along here, the particles will have some reasonably well-defined momentum because their energy will consist of their kinetic energy. So, along here we will have that that the amplitude is going to be on the order or e to the i, p upon h-Bar times x.

So that's a reasonable model for what the, how the amplitude varies with position and that's the new information that we bring to bear on this. So what will be the difference? So the probability to arrive at x as you recall, was equal to the amplitude to arrive by the top slot plus the amplitude to arrive at the bottom slot mod squared. And the argument that this was a plus mod squared plus a minus mod squared plus twice the real part of a plus a minus.

And these are about equal and well, these were the classical probabilities so these were p plus plus p minus. And then we have this quantum interference term which we want to assess, which... So what is this quantum interference term? So the interference term is going to be mod a plus mod a minus neither not very interesting and the crucial thing is that this one is going to be e to the i, p upon h-Bar d plus.

And the other one is going to be e to the minus i, p upon h-Bar, sorry plus, sorry. One of these needs to have a star in it which means that one of these requires a minus sign. So we're interested, excuse me in the real part of this. And the real part, so this thing can be written as cos p upon h-Bar, d plus d minus, minus d plus etc, etc, etc. So this is looking like two, the probability to go either way, times, through each hole separately, times the cosine of p over h-Bar, d plus minus d minus. Actually it's easier to do it the other way round isn't it – minus plus.

So, what is this difference of distances? This difference of distances from up there, well let's, binomial expand this and, because we can argue that l is going to be big experimentally compared to x. x will be millimetres; l will be a metre or so. So we can binomial expand this and say that this is l. It's about a half of l brackets one plus x minus s squared over l squared plus dot, dot, dot. And the other one's going to be about half l one plus x plus s squared over l squared plus dot, dot, dot.

So when we take the difference of those two, so this is going to be 2 p plus minus this probability times the cosine p upon h-Bar of, the difference is going to be x plus s squared over l minus x minus s squared over 2 l in fact. So when you take the difference of those two you will be looking at 2 x s over l. Yes, 2 s x over l is what that difference will be.

So, now let's put in, now we need to put in some numbers, right. Suppose we take the energy which is p squared over 2 m. What do want to do? We want to, no. so what does this do? This gives us a probability of arrival as a function of x which is going to consist of a, of twice the sum of these two which is going to be about equal. And so about twice this plus twice this thing times this cosine. So it's going to be an oscillating commodity which will be doing this. And there's some characteristic distance between these minima, which so this, we'll call this delta, x say, no let's call it big X, that's what the…

So we'll call this big X, the distance between the places where it's a minimum because the quantum interference term is cancelling the classical term. And this difference is what causes that, the argument of that cosine to become 2 Pi. So we can write 2 Pi is equal to p over h-Bar times 2 s over l times x. In other words we have a formula for the distance between the minima which is 2 Pi h-Bar, but h-Bar is h Planck's constant over 2 Pi so that's going to be h over p, h l, sorry, yes, of s. Yes.

And we could also write this I guess, we could also say that, oh no let's not bother.

So, let's put some numbers in. let's say the e the energy is, so in order to get a big value of x we want to take a big value of l needless to say we want to take a small value of p and of course a small value of s.

Male And another 2 on the bottom, is that…

Contributor I've lost a 2, yes you're quite right. So, let's take l to be a metre; let's take s to be, I think, oh dear what did I do? I think it's, yes a micron because you want to make it as small as you can but if you make it much smaller than a micron you'll find it difficult to make the whole, using ordinary materials. And let's, in order to get a small value of p we want to take a small value of the energy, but you can't take too small a value of the energy or your particles will be deflected by stray electromagnetic fields and stuff. And it'll be difficult to keep any kind of coherence.

So let's take a 100EV say, as a sort of low speed. If you plug all this stuff into there so that gives you, what does it give you? A twentieth of a millimetre or something I think it's, is it 0.06 millimetres, which is obviously perfectly, perfectly observable. Yes. So this, such an interference experiment is possible but is hard using electrons.

If you do the same thing with bullets, when we're not expecting anything to happen what could we do? We would take the velocity from a gun is say 300 metres a second; it might be a bit faster these days, I'm not sure but that's a classical, that's you know, faster than sound so that's sort of a reasonable ballpark figure. Suppose we took l to be one kilometre, a thousand yards a pretty reasonable shooting distance for a rifle. And if we took the mass to be 10 grams, put it into the same formula and we discover that x is some ridiculous figure, 10 to the minus 29 metres.

So it's obvious that you cannot observe this interference using anything like a bullet, any kind of macroscopic, any kind of macroscopic object because it's going to be vastly bigger itself than the size of the interference pattern. Obviously an absolutely basic requirement for this experiment to work, is the physical size of your particle has to be smaller than the value of x that you'd derive out of this so you haven't a hope of measuring this interference.

So that's why classically we don't, we're unaware of this interference term. But I would remind you that in the last lecture we recovered classical results, which explained why cricket balls move as they do, why satellites and so on move as they do. By interpreting, we calculated this, we obtained results which recovered classical physics by decomposing the amplitude, to arrive into a sum of contributions from states of different well-defined momentum. And these were all interfering with each other and the classical physics came back as a result of quantum interference.

So this quantum interference on the one hand which is something which is very hard to observe with classical objects. On the other hand our entire picture of the classical world, a classical world is only recovered through quantum interference. It's not some esoteric corner of the subject but it is hard to, it's hard to have it happen in a controlled way.

Okay. So, we, yes, we should just, so we've done the position representation in just one dimension. Everything is being, you know one dimensional motion along x. We obviously need to generalise the position representation to three dimensions because we live in a three dimensional world for whatever reason. And the generalisation is nice and trivial. We don't need to worry about it. we have, we now have three position operators x, y and z. Also known as x i, all right and we have of course three momentum operators, three more operators p x, p y and p z also known as p i.

And we have that every one of these operators commutes with the other one so we have that x i comma x j is nothing and every one of the momentum operators commutes p i comma p j equals nothing so it is possible to simultaneously know your x coordinate, your y coordinate and your z coordinate. There's a complete set of eigen states of well-defined states, of states where you know all those three in simultaneously. Or you can know all three components in momentum but you can't know, there's not a complete set of states for knowing and so on.

And the only other interesting thing we have to have is x i commuted with p j is i h-Bar delta i j. So it is possible to know the x position and the y momentum but it's not possible to know the x position and the x momentum. So most of these operators commute with, well each operator commutes with five of the, sorry four of the remaining five operators but it does not commute with its own momentum. That's what it, each of these position operators. So that's the generalisation there. What else do we have to say? Where we used to have a wave function of psi being a function of scalar x, we now it's trivial the argument of the wave, we can now label a complete set of states by x, y and z.

So we can write that there's a, we have states of well-defined position which are labelled by a vector now, vector position x because there are three, this is an eigen state of the x operator. It's an eigen state of the y operator and an eigen state of the z operator. So we need three eigen values written inside here to describe what this is. It is, that's at a mathematical level. At a physical level this is the state of being at the location position vector x. Correspondingly our wave functions become functions of x, y and z because they become these complex numbers, right.

That's still a complex number, this complex number but it's a function now of x and y and z for the locations of the particles. Similarly we have states of well-defined momentum u p, we have states, yes, yes. u p of x which is x p. So, now we have, here we have p x, p y and p z because we have a state of well-defined momentum, which is labelled with all three components in momentum.

So we have this function of a complex of three complex, sorry, this complex function of three variables x, y and z labelled by the momentum. This is just an identical notation.

Whereas in single, when we were doing this in one dimension we found that this was e to the i, p over h-Bar times x not vectors. Now that's a vector, that's a vector and whereas on the bottom we used to have h-Bar to the one half, now we have h-Bar to the three halves reflecting the fact that there's an x component to this, a y component to this and a z component to this.

So this, this wave function of the state of well-defined momentum has now become a plain wave whose wave surfaces are normal to the vector p. And that's what it is. It's easy to check that that stuff works. It's a very straightforward generalisation of what we did before.

And I think that's all we have to say, oh not quite. We also want to say what the momentum operator p looks like. So previously we had that x, p, x p sorry, not what I'm talking about it, yes. x p of psi was minus i h-Bar was introduced by this formula here dbdi the x of x of psi. All right, that was what we did in one dimension. That generalises in three dimensions very straightforwardly to x p psi. So, that's become a vector, that's become a vector because we have to write down what it is for p x, p y and p z. This is really going to be a shorthand for three formulae. And it's going to be minus i h-Bar, gradient operator on the function of three variables, functional space this one here, right, the wave function.

So, this is a vector reflecting the fact that that's a vector. This is just a label which appears on both sides of the equation. That's what this, this formula generalises to that formula. I don't think we need be detained about that any longer.

Before we leave the position representation it's good to do a useful result which falls into our laps now because of work we've already done, called a virial theorem which is a, it's a resulting classical physics which you may not have met, I don't know. But in a way should have met, have you, did you cover the virial theorem in classical mechanics, anywhere? No, anyway, so it's, there's nothing quantum mechanical about the virial theorem. It has a classical counterpart. But it's going to fall into our laps because we've got this powerful machinery.

So, do you recall if we are in a stationary state, that is to say a state in which the results of measuring energy are certain, then all expectation values for such a state are constants. That's why we call it a stationary state. It's going nowhere. So every expectation value for a stationary state, for a state or well-defined energy is independent of time.

So we want to exploit that result. So for a stationary state, this is just recalling what we already had. It was a consequence of Ehrenfest's Theorem for a stationary state we have that dbdi time of e q e equals nought for all operators q. It doesn't matter what observer you stuff in there, so long as the observer will, is defined in a way that is independent of time. So it's something like position, momentum, angular momentum, whatever. It has a vanishing rate of change with respect to time. It's a constant.

So we now apply this result to q is equal to x dot p. So then we have that nought is equal to dbdi t, sorry. Sudden moment of doubt. Yes, yes. So I want to apply this to x dot p e and let's divide, oh stick in an i h-Bar. That by Ehrenfest's Theorem is x dot p comma h. whoops. So Ehrenfest's Theorem tells us that this rate of change which vanished is equal to this here. And now let's take, suppose we're dealing with a particle which has, which has kinetic energy and potential energy so we'll take the Hamiltonian to be of that form, which is a pretty useful form. And stuff it in there and we're going to have that nought is equal to e x dot p comma p squared over 2 m plus v close square brackets.

So now we need to work out what this commutator is and this is where a little bit of, so this is where we get a bit of practice in using the three dimensional generalisation. We obviously have two things to work out. We've got a commutator of x comma p with p squared so let's work that out. x comma p with p squared. Now we write that in components, we write x dot p sorry, x dot p, did I say comma? x dot p comma p squared. This x dot can be written as a sum over j, j equals one to three of x i p i, sorry x j p j. So that's just a way of writing that. And now I have a sum p k, well p squared k. So now I'm summing over k as well.

Right, p squared is p x squared plus p y squared plus p z squared. Now we can work out this using our rules for a commutator. We had that rule that a comma b, sorry a b comma c was equal to a comma c, b plus a b comma c. p, we know that p k commutes with p j. That's been written down up there so that commutator vanishes. That's this one here in some sense. Sorry, that's this one here in some sense.

And so what we're left with is, so we have this double sum, we're going to have x j p k, p j, sorry that's squared, squared comma, no no, no comma. So that's what we get so this has to be commuted with that. That's what I've written down I hope and then there should in principle be another term this commuting with this and that vanishes because p j commutes with p k for all j and k.

So we have to work out what this one is now. And we can use the same rule is we're being pedantic we would say this is x on p k p k. So we would say that this is x j p k p k plus p k the commutator of x and p k. I'm using the same rule and that all has to be multiplied by p j. The same, because I'm now writing p squared as p k p k. But this is i h-Bar. This is i h-Bar so these two terms actually contribute the same thing. This becomes 2 i h-Bar p times delta j k times p k times p j. And I'm sorry I've lost track of the sum sign. Here we have a sum sign. We're summing over j and we're summing over k.

Sum over j and you get nothing because of this delta j k unless j is equal to k. So this becomes p k p k summed over k. But p k p k summed over k is the same thing as p squared. So this is 2 i h-Bar p squared. That's what the commutator is of x dot p with p squared.

Now let's write, let's do the x dot p commutator with v which is itself a function of x of course. These things all ought to have hats really but one gets, it's difficult to write down enough things. Well, what we want to do is write this thoroughly in the position representation. In the position representation x dot p is minus i h-Bar x dot gradient. Right, that's what this becomes in the position representation on v which becomes a function of x, just so this is in the position representation. So what does that mean? That means i h-Bar, minus i h-Bar brackets x dot gradient working on v minus v x dot gradient.

And this is an operator statement so it's waiting for you to put in the function of your choice of psi on the right, right? There's a virtual function there for it to work on, that's the meaning of the v x dot gradient. And this x dot gradient v doesn't mean x dot gradient of only v. It means that everything that is to the right of it including `[[your 0:27:52]]` upsi. So when you use this x dot v on v times alone, you'll get a term and then you will still have to use the x dot v on upsi but the result of using the x dot v on the upsi will be killed by here, an x dot v on upsi.

So what this is equal to is minus i h-Bar x dot gradient of v. That's all that survives this is the action of the nabla, the gradient operator on the potential itself. The operation of, the action of the gradient operator on the wave function that's virtually sitting here is cancelled by this contribution here.

So we now have, we can put these results back into what we had up there. So what we had was nought is equal to, yes, is equal to the sum of these commutators, is equal to e x dot p comma p squared e plus e x dot p v. That's just summarising where we stand. This we've discovered to be, this is 2 i h-Bar, this commutator turned out to be p squared so it becomes the expectation, oops there should have been over 2 m on this shouldn't there, because it was the Hamiltonian p squared over 2 m. Yes, this p came from the Hamiltonian where it was p squared over 2 m, this v came from the Hamiltonian where it was just v.

So we have over 2 m, no let's leave that alone, of p squared over 2 m. e plus, we figured out that this one was minus i h-Bar. So we want to cancel what we, this is the expectation value of the kinetic energy, clearly right. p squared over 2 m is the kinetic energy. This is the expectation value of it. So cancelling the i h-Bar we can say that 2 times the expectation value of the kinetic energy is equal to this stuff.

That's as far as we can go in general. But consider now very important cases have that v of x is proportional to mod x to the alpha. So, for example for a simple harmonic oscillator we're about to discuss, the potential energy goes like x squared, alpha's 2. If we were dealing with a Coulomb interaction the potential energy goes like 1 over radius. So it would be v of r is proportional to 1 over r. Well, this mod x is r so alpha would be minus 1. So we can say that alpha equals 2 in simple harmonic motion, alpha equals minus 1 is Coulomb.

There are, you know you can think of other power laws which are relevant. In this case, so then, if we ask ourselves what is x dot gradient of v. Well, that's going to, so we'll say that this is equal to some constant a times x to the alpha, what is this going to be? It's going to be alpha mod x to the alpha minus 1 times x dot the gradient of mod x. And the gradient of mod x is, the gradient of mod x is x, the unit vector x so it's the vector x over mod x, so this is equal to, sorry this is a. We have a x to the alpha minus 1 times x dot x over mod x. So here this mod x is going to make this an alpha to the minus 2.

But from this x dot x we're going to get a mod x squared so this is going to be and I've lost, sorry this was an alpha. There was also an a unfortunately, yes sorry. We need an a and an alpha. This is going to be alpha times a x to the alpha which is alpha times v. So if v has a power law dependence on distance from the origin, then x dot grad v is simply alpha times v.

So when we put this result back into that formula, back into this statement here we have that twice the k e expectation value is equal to alpha times the expectation value of the potential energy. So that's our Kepler formula. In the case of simple harmonic motion alpha is 2 and kinetic energy is equal potential energy. In the case of Coulomb interaction, where alpha is minus 1, you have that the potential energy is minus twice the kinetic energy, which is to say that the particle has lost 2 units of energy, falling in from infinity into a bound orbit it's lost 2 units of energy. 1 unit's been sent off to infinity and radiation or something, and 1 unit is used as kinetic energy of its orbit. So that's the, this is the Virial Theorem.

So now, we open a new chapter as it were by talking about harmonic motion. The harmonic oscillator is the single most important dynamical system in physics. Most of field theory, most of condensed, quantum field theory, most of condensed matter physics is fiddling with more or less with harmonic oscillators which are decorated in some ways. So the basic physics is that of the harmonic oscillator and it's worth just taking a moment to understand why harmonic oscillators are all over the place, the universe.

Physicists, a fundamental position of physicists, what physicists like to represent the universe is a collection of harmonic oscillators. And this is partly because physicists are maybe brighter than some other people but they're still pretty stupid. We have quite a small bag of tricks. And anharmonic oscillator is a trick that we have and it's an incredibly useful trick for this reason, that if you plot force in some direction versus displacement from a point of equilibrium, you will get a curve which does something like this.

The force vanishes at equilibrium. At a point of equilibrium of a system the force on it obviously vanishes so if you do a plot of forces versus distance you'll get a curve something like this, passing through zero at the point of equilibrium which I happen to have put at the origin of x. but, you know that's by construction clearly. And the general idea is that most of the time you can, sorry that's meant to go through the origin, most of the time you can represent this to some, to a good approximation you can say that f of x is about equal to k x plus order of x squared or whatever.

So to lowest order approximation, because f has to vanish at a point of equilibrium in the neighbourhood of equilibrium, f is going to be proportional to x. And if f ix, if we neglect this, if this is small then we have harmonic motion for displacements, these small displacements around here. So this is why harmonic oscillators are ubiquitous, and incredibly valuable model we can apply, we can use to understand many, many systems. Because many systems for small displacements almost all systems for small displacements look like harmonic oscillator.

Okay. So let's agree what the Hamiltonian of this thing should be. The Hamiltonian of our harmonic oscillator should be p squared over 2 m plus a half k x squared, right, because if the force is going like that you integrate it out. This becomes the potential energy and this is of course the kinetic energy, we're familiar with that already. It's better though to write this in a different way to anticipate results that are to come, and to write this as p squared plus m omega x squared over 2 m. So, and of course omega squared is k over m.

So it's easy defining omega squared to be k over m. It's easier to write this formula like that and that's how I want to write it. Should you want to reproduce this formula, just think about dimensional analysis. We want to have p squared over 2 m because it's the kinetic energy, we're always saying that. And here I want something that's proportional to x squared and has the dimensions of momentum. And obviously omega x has dimensions of speed so m omega x has dimensions of momentum so that's why, you know, that enables you to recover that quickly from this. And that's the way to go for practical purposes.

So there's our Hamiltonian. And we're trying of course to solve, the stationary states are the key to understanding dynamics because they have this trivial time evolution. And by decomposing any initial condition into a sum of stationary states into a linear superposition of stationary states, then evolving the stationary states we find out how any arbitrary initial condition evolves in time. So that's why we want these stationary states. I've said that before and I'll say that again.

So, we want to find states of well-defined energy. This is the problem we want to solve. And this is a completely generic situation in physics. First of all you think about your physical system. On the grounds on physics you write down the Hamiltonian. Then the next thing you do is you find the damn stationary states because once you've got those you can do anything you want, pretty much.

So that's what we're trying to solve. The way to do this, the proper way to find these states, so we need to find the energies that are possible and we need to find the corresponding states. And the way to do this is to introduce some new operators. Let's introduce a which is m omega x plus i p over the square root of 2 m omega, or h-Bar omega.

Why do I write that down? Well, basically because I know where I'm going but just to give you some sense of direction. The general idea here is that we want to factorise that. That's the general idea. We want to factorise the Hamiltonian into, you know it's a quadratic expression. It seems kind of reasonable to factorise it. If these were, if these weren't operators, because these are operators, sorry and in future I'm not going to even attempt to put hats on operators, right. These are operators despite the absence of hats. It's just too difficult to remember to put the hats on and takes too much time, and grown-ups never do.

But these are operators. Now but if they weren't operators this and its complex conjugate would factorise that. So that's the drift, okay. Let's write down its, well its complex, this of course is an operator and it's not an observable. It's not a Hermitian operator. It's, what is its dagger, a dagger, its Hermitian adjoint is this thing dagger which is itself because x is a Hermitian operator on its own dagger. So it's m omega x plus this thing dagger. p is its own dagger but i has, the dagger of i, the Hermitian adjoint of i is minus i so this is minus i p over, of course this on the bottom is a real number so it's its own, it's its own complex conjugate.

So here we have two operators and the general idea is they're going to factorise h or they almost do or whatever. That's the plan. And this is called an annihilation operator and this is a creation operation. And we'll, the reason they have these names will emerge but it is that if you use this on a state, this operator increases the excitation of our harmonic oscillator and this oscillator reduces the excitation of our harmonic oscillator. And since in quantum field theory, particles are excitations of the vacuum this thing creates a particle because it creates an excitation which is a particle, and this thing destroys the particle because it destroys an excitation.

So what we next do is work out what a dagger a is because the idea was that this product would be more or less the Hamiltonian. So what exactly is it? Let's get this right. m omega x minus i p. m omega x plus i p over 2 m h-Bar omega. Now when we write this out we have the obvious terms. We have p squared and we have m omega x squared. So let's write those down. That's p squared plus m omega x squared all over 2 m h-Bar, h-Bar omega 2 m omega h-Bar, whatever.

And then we have some additional terms which would cancel in classical algebra but don't now, because we have an x, we have an m omega x times i p and here we have an m omega x on the, sorry we have an i, minus i p times m omega x. So the additional term is an m omega x, m omega i x comma p. And it's again over 2 m h-Bar omega. Right, so this is the Hamiltonian over h-Bar omega. And this is an i h-Bar and the i(s) make a minus 1 with this so this is going to minus a half and everything else will cancel. Right. Because we'll, we've got an m omega here and we're going to get an h-Bar from there so the rest cancels.

So, I should have explained, sorry. I wanted to factorise this and this on the bottom, this normalising factor on the bottom is put in. It's not really essential but it's very convenient. And it's put in in order to make this dimensionless.

So, just to check that that's true, h-Bar has dimensions of position times momentum, right. So it has the dimensions of position times momentum so what we have here is m x, sorry m omega x, which we've agreed has dimensions of momentum, times p which has dimensions of momentum. And then we take the square root. So this on the bottom has dimensions of, the whole square root of momentum and therefore cancels dimensions of what's on the top. So it's dimensionless. That's the purpose of the, that's the purpose of the horrible square root.

So we find that this product which is dimensionless is equal to the Hamiltonian divided by h-Bar omega which has the dimensions of energy, because h-Bar also has the dimensions of energy times time, omega of course has the dimensions of one over time. It's the frequency of the oscillator, so this is, has dimensions of energy minus a half, which is obviously dimensionless.

So we have indeed, almost factorised. We have a statement now that h can be written as h-Bar omega which carries the dimensions times a dagger a plus a half. We've almost factorised h, just as that there.

The next thing we want to do is calculate the commutator a dagger comma a, yes we've just got time to do this, a dagger a of these two operators. So of course we will a one over two m h-Bar omega as a factor on the bottom because each of these a(s) brings in its own square root. And then we will have the commutator of m omega x minus i p on m omega x plus i p.

Now, we have, this breaks down in to four commutators in principle. There's the commutator of this with this. And the commutator of this with this. The commutator of this with this obviously vanishes because x commutes with itself. And the commutator of this with this is, so we're going to have an m omega i times x comma p. That's the commutator of this with this. And now we have to deal with these, with these terms. This produces a non negligible commutator with that. We're going to have minus m omega i times p comma x. And then we'll have the commutator of p with itself which will vanish.

If I swap those two over, then clearly I change the sign in front and then this becomes a plus x comma p. It becomes this thing all over so that cancels this. And this whole caboodle is going to equal i x comma p over h-Bar, because we're going to get a 2, these two terms are going to add together to make a 2 which cancels with this, and the m omegas clearly go. x comma p is itself equal to i h-Bar so the i(s) make a minus one, the h-Bars cancel and this is equal to minus one. So these two operators have non vanishing commutator actually equal to minus one.

Yes, well we seem to still have time to nail this problem I think. So, let us suppose we have got a state of, a stationary state. Let us now apply the operator a dagger to both sides of this equation, right. And this is just an eigen value; it's only a number so I can then write e a dagger e is equal to a dagger h e. That's obvious. I would like to swap these over so I jolly well do. I say this is equal to h a dagger plus a dagger commutator h. So this commutator puts in what I'm supposed to have and takes away what I'm not supposed to have but have previously written down.

But we know what h is in terms we know that h is equal to, there it is h-Bar omega a dagger a, so let's use that. So this is h a dagger plus commentator of a dagger and h turns out to be a dagger a plus a half, close brackets, h-Bar omega to carry the dimensions, close that, close that and stick in our e that we first thought of.

So all I have done is replaced h by an expression we already derived, yes. Now I have to take the commutator of a dagger with this, and with this. The commutator of a dagger with the half clearly vanishes because a half is just a number, not an operator. The commutator of a dagger with itself vanishes so when we do the commutator with this product, there should in principle be two terms but only one of them survives. And that term is, that term is this sticks, this stands idly by while the a dagger works on that. And then I have an h-Bar omega, h-Bar omega, close brackets, close brackets, e.

But we've just worked this thing out and found that it's minus one, right, a dagger a turned out to be minus one so this is equal to h a dagger e minus h-Bar omega a dagger e. And just to remind us what we had on the left, what we had on the left was e a dagger e. This is just a restatement of what's been at the top. So we take this a dagger e and we obviously join it on to that a dagger e and we discover that h on a dagger e is equal to, h working on this the kept you get by using a dagger on e, is equal to e plus h-Bar omega of a dagger working on e.

What does this tell us? It tells us that we have, out of the state which had energy e, we have constructed a state by multiplying by a dagger which has energy e plus h-Bar omega. So this means that a dagger e is equal to a constant, normalising constant not discussed, times e plus h-Bar omega. A new stationary state. This is an incredibly powerful result because it immediately follows that we have states, if we can find a state e we can immediately generate e plus h-Bar omega by using this a dagger beast. And also, if we use a dagger on this it follows that we're going to get e plus 2 h-Bar omega and we're going to get another, if we use a dagger on this we're going to get e plus 3 h-Bar omega.

So we're going to get a whole infinite series of states of ever increasing energy simply by applying a dagger again, and again, and again. So what remains is to find what e, what number e is and that we will do first thing tomorrow.