The eccentric genius of Lewis Carroll

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Outline of talk

- Why voting procedures are more problematic than they seem
- *Two* eccentric misunderstood mathematicians:
  - CLD (Lewis Carroll, 1832-98)
  - Duncan Black 1908-91. But for Black, nobody would have understood CLD's originality
- CLD's contribution to axiomatic theory
- CLD's contribution on proportional representation
- What is it about voting theory that attracts eccentric mathematicians?

Why voting procedures are more problematic than they seem

- Central 'paradox': with >2 voters and >2 options, a majority-rule *cycle* (CLD originated the term) is always possible

- Two directions:
  - towards impossibility (Arrow);
  - Towards existence results (May, Black)

- Choice and PR systems and their properties: Condorcet, Borda, STV

The rules

- Condorcet: *Make exhaustive pairwise comparisons. Select the candidate (option) that wins every time*
  - Problem: may not exist
- Borda: *Score each candidate at n-1 for first place down to 0 for last place. Add up scores*
  - Problem: violates independence axioms
- STV: elimination method
  - Problem: violates both Condorcet-efficiency and independence

The minimum cyclical case

	Voter 1	Voter 2	Voter 3
Best	a	b	c
Middle	b	c	a
Worst	c	a	b

Theorems

- With ≥ 2 voters and ≥ 3 options, no decisive aggregation function can satisfy ***U***, ***P***, ***I***, and ***D*** (Arrow 1951/63)

- Simple majority rule uniquely satisfies ***anonymity***, ***neutrality***, & ***+ve responsiveness*** (May 1952)

- Single-peakedness guarantees acyclicity (Black 1948, 1958)

Theorems (cont)

- All social choice rests on these 3 theorems: one impossibility result, one existence result, and one uniqueness result.

- From the impossibility result, we can stop looking for the best electoral system. It doesn't exist

- From the existence results, we can define the circumstances in which the problems rediscovered by CLD don't arise

CLD's main works

- *A Discussion of the various methods of procedure in conducting elections* 1873
- *Suggestions as to the best method of taking votes, where more than two issues are to be voted on* 1874
- *A method of taking votes on more than two issues* 1876
- *Lawn tennis tournaments* 1883
- *The principles of parliamentary representation* 1884

The strange career of Duncan Black

- Long before discovering CLD, aimed for "pure science of politics"
- Proved his theorem, and discovered cycles, during WWII firewatching
- Published (to no understanding) 1948, 1951
- Then discovered CLD: and that they had both independently rediscovered the work of Borda and Condorcet in C18 France
- His book on CLD and PR published posthumously, edited from his papers by IM and collaborators

How did Black show that CLD's discoveries were independent of Condorcet and Borda?

- By checking the Bodleian and Ch Ch library copies of their work

CLD's contribution to axiomatic theory

- Rediscovered and characterized Borda rule
- Then changed his mind on discovering its Condorcet-inefficiency
- Finally proposed the hybrid Dodgson rule
- First to use the word cycle and to use matrix notation

CLD's contribution on proportional representation

- Originally, even Black found it "trivial"
- It isn't
- Characterizes properties of systems with n seats to fill, each voter has m votes, $m < n$
- In limit, Single Non-Transferable Vote
- Practical importance in his time: the Cairns system and his correspondence with Lord Salisbury

What is it about voting theory that attracts eccentric mathematicians?

- – Ramon Llull c 1235-1315 – "the mad doctor"
- – Nicolaus Cusanus 1401-64
- – Marquis de Condorcet 1743-94
- – J.-C de Borda 1733-99
- – E.J. Nanson 1850-1938
- – Duncan Black 1908-91
- Nobody (or almost) understood what they were talking about. ?The perils of common sense.